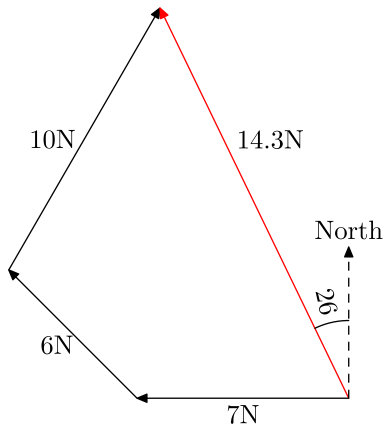
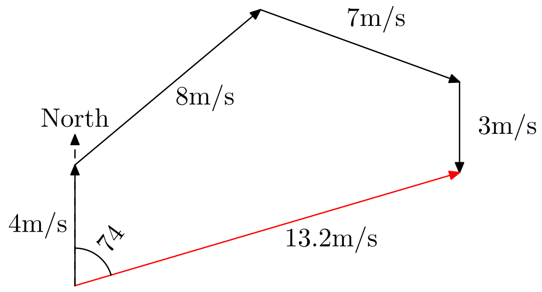
Chapter 4

Exercise 4A

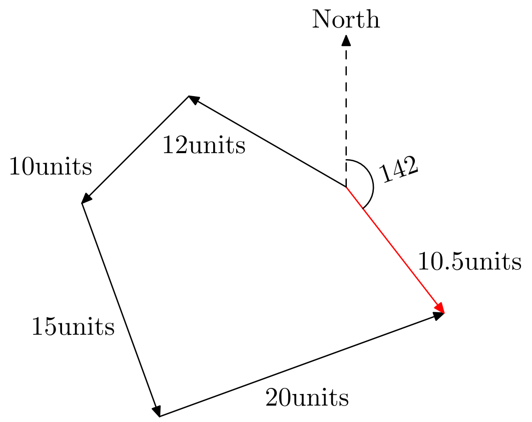
1.



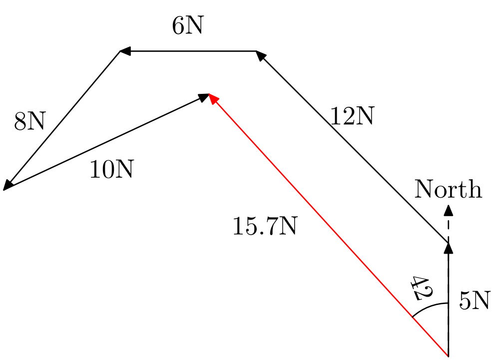
2.



3.



4.



5. $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j}$;
$\mathbf{b} = 3\mathbf{i} + 1\mathbf{j} = 3\mathbf{i} + \mathbf{j}$;
$\mathbf{c} = 2\mathbf{i} + 2\mathbf{j}$;
$\mathbf{d} = -1\mathbf{i} + 3\mathbf{j} = -\mathbf{i} + 3\mathbf{j}$;
$\mathbf{e} = 0\mathbf{i} + 2\mathbf{j} = 2\mathbf{j}$;
$\mathbf{f} = -1\mathbf{i} + 2\mathbf{j} = -\mathbf{i} + 2\mathbf{j}$;
$\mathbf{g} = 1\mathbf{i} - 2\mathbf{j} = \mathbf{i} - 2\mathbf{j}$;
$\mathbf{h} = 4\mathbf{i} + 0\mathbf{j} = 4\mathbf{i}$;
$\mathbf{k} = 2\mathbf{i} - 4\mathbf{j}$;
$\mathbf{l} = 4\mathbf{i} - 1\mathbf{j} = 4\mathbf{i} - \mathbf{j}$;
$\mathbf{m} = -4\mathbf{i} - 1\mathbf{j} = -4\mathbf{i} - \mathbf{j}$;
$\mathbf{n} = 9\mathbf{i} + 2\mathbf{j}$;

6. $|\mathbf{a}| = \sqrt{3^2 + 2^2} = \sqrt{13}$;
$|\mathbf{b}| = \sqrt{3^2 + 1^2} = \sqrt{10}$;
$|\mathbf{c}| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$;
$|\mathbf{d}| = \sqrt{1^2 + 3^2} = \sqrt{10}$;
$|\mathbf{e}| = 2$;
$|\mathbf{f}| = \sqrt{1^2 + 2^2} = \sqrt{5}$;
$|\mathbf{g}| = \sqrt{1^2 + 2^2} = \sqrt{5}$;
$|\mathbf{h}| = 4$;
$|\mathbf{k}| = \sqrt{2^2 + 4^2} = 2\sqrt{5}$;
$|\mathbf{l}| = \sqrt{4^2 + 1^2} = \sqrt{17}$;
$|\mathbf{m}| = \sqrt{4^2 + 1^2} = \sqrt{17}$;
$|\mathbf{n}| = \sqrt{9^2 + 2^2} = \sqrt{85}$;

7. $|(-7\mathbf{i} + 24\mathbf{j})| = \sqrt{7^2 + 24^2} = 25$Newtons

8. (a) $(5\cos(30°)\mathbf{i} + 5\sin(30°)\mathbf{j})$units
$\approx (4.3\mathbf{i} + 2.5\mathbf{j})$units

(b) $(7\cos(60°)\mathbf{i} + 7\sin(60°)\mathbf{j})$units
$\approx (3.5\mathbf{i} + 6.1\mathbf{j})$units

(c) $(10\cos(25°)\mathbf{i} + 10\sin(25°)\mathbf{j})$units
$\approx (9.1\mathbf{i} + 4.2\mathbf{j})$units

(d) $(7\sin(50°)\mathbf{i} + 7\cos(50°)\mathbf{j})$N
$\approx (5.4\mathbf{i} + 4.5\mathbf{j})$N

(e) $(5 - 8\cos(60°)\mathbf{i} + 8\sin(60°)\mathbf{j})$m/s
$\approx (-4.0\mathbf{i} + 6.9\mathbf{j})$m/s

(f) $(10\cos(20°)\mathbf{i} - 10\sin(20°)\mathbf{j})$N
$\approx (9.4\mathbf{i} - 3.4\mathbf{j})$N

(g) $(-4\cos(50°)\mathbf{i} + 4\sin(50°)\mathbf{j})$units
$\approx (-2.6\mathbf{i} + 3.1\mathbf{j})$units

(h) $(8\cos(24°)\mathbf{i} - 8\sin(24°)\mathbf{j})$units
$\approx (7.3\mathbf{i} - 3.3\mathbf{j})$units

(i) $(-6\sin(50°)\mathbf{i} - 6\cos(50°)\mathbf{j})$units
$\approx (-4.6\mathbf{i} - 3.9\mathbf{j})$units

(j) $(-10\cos(50°)\mathbf{i} + 10\sin(50°)\mathbf{j})$m/s
$\approx (-6.4\mathbf{i} + 7.7\mathbf{j})$m/s

(k) $(-8\cos(25°)\mathbf{i} - 8\sin(25°)\mathbf{j})$N
$\approx (-7.3\mathbf{i} - 3.4\mathbf{j})$N

(l) $(5\cos(35°)\mathbf{i} + 5\sin(35°)\mathbf{j})$m/s
$\approx (4.1\mathbf{i} + 2.9\mathbf{j})$m/s

9. (a) $|\mathbf{a}| = \sqrt{3^2 + 4^2} = 5$

$$\theta = \tan^{-1}\frac{4}{3} \approx 53.1°$$

(b) $|\mathbf{b}| = \sqrt{5^2+2^2} = \sqrt{29}$

$\theta = \tan^{-1}\frac{2}{5} \approx 21.8°$

(c) $|\mathbf{c}| = \sqrt{2^2+3^2} = \sqrt{13}$

$\theta = 180° - \tan^{-1}\frac{3}{2} \approx 123.7°$

(d) $|\mathbf{d}| = \sqrt{4^2+3^2} = 5$

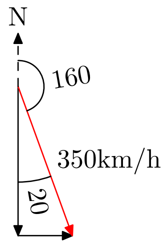
$\theta = \tan^{-1}\frac{4}{3} \approx 53.1°$

(e) $|\mathbf{e}| = \sqrt{5^2+4^2} = \sqrt{41}$

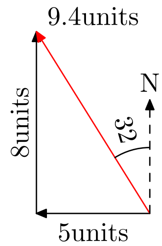
$\theta = \tan^{-1}\frac{4}{5} \approx 38.7°$

(f) $|\mathbf{f}| = \sqrt{4^2+4^2} = 4\sqrt{2}$

$\theta = \tan^{-1}\frac{4}{4} = 45.0°$

10. 

Northerly component $= 350\cos 160°$ (or $-350\cos 20°$) $= -328.9$km/h
Easterly component $= 350\sin 160°$ (or $350\sin 20°$) $= 119.7$km/h

11. 

Magnitude$= \sqrt{5^2+8^2} = \sqrt{89} \approx 9.4$units
Direction$= 360 - \tan^{-1}\frac{5}{8} \approx 360 - 32.0 = 328°$T

12. (a) $\mathbf{a}+\mathbf{b} = 2\mathbf{i}+3\mathbf{j}+\mathbf{i}+4\mathbf{j} = (2+1)\mathbf{i}+(3+4)\mathbf{j} = 3\mathbf{i}+7\mathbf{j}$

(b) $\mathbf{a}-\mathbf{b} = (2-1)\mathbf{i}+(3-4)\mathbf{j} = \mathbf{i}-\mathbf{j}$

(c) $\mathbf{b}-\mathbf{a} = (1-2)\mathbf{i}+(4-3)\mathbf{j} = -\mathbf{i}+\mathbf{j}$

(d) $2\mathbf{a} = 2(2\mathbf{i})+2(3\mathbf{j}) = 4\mathbf{i}+6\mathbf{j}$

(e) $3\mathbf{b} = 3(\mathbf{i})+3(4\mathbf{j}) = 3\mathbf{i}+12\mathbf{j}$

(f) $2\mathbf{a}+3\mathbf{b} = (2\times2+3\times1)\mathbf{i}+(2\times3+3\times4)\mathbf{j} = 7\mathbf{i}+18\mathbf{j}$

(g) $2\mathbf{a}-3\mathbf{b} = (4-3)\mathbf{i}+(6-12)\mathbf{j} = \mathbf{i}-6\mathbf{j}$

(h) $-2\mathbf{a}+3\mathbf{b} = (-4+3)\mathbf{i}+(-6+12)\mathbf{j} = -\mathbf{i}+6\mathbf{j}$

(i) $|\mathbf{a}| = \sqrt{2^2+3^2} = \sqrt{13} \approx 3.61$

(j) $|\mathbf{b}| = \sqrt{1^2+4^2} = \sqrt{17} \approx 4.12$

(k) $|\mathbf{a}|+|\mathbf{b}| = \sqrt{13}+\sqrt{17} \approx 7.73$

(l) $|\mathbf{a}+\mathbf{b}| = |3\mathbf{i}+7\mathbf{j}| = \sqrt{3^2+7^2} = \sqrt{58} \approx 7.62$

13. (a) $2\mathbf{c}+\mathbf{d} = (2+2)\mathbf{i}+(-2+1)\mathbf{j} = 4\mathbf{i}-\mathbf{j}$

(b) $\mathbf{c}-\mathbf{d} = (1-2)\mathbf{i}+(-1-1)\mathbf{j} = -\mathbf{i}-2\mathbf{j}$

(c) $\mathbf{d}-\mathbf{c} = \mathbf{i}+2\mathbf{j}$

(d) $5\mathbf{c} = 5\mathbf{i}-5\mathbf{j}$

(e) $5\mathbf{c}+\mathbf{d} = (5+2)\mathbf{i}+(-5+1)\mathbf{j} = 7\mathbf{i}-4\mathbf{j}$

(f) $5\mathbf{c}+2\mathbf{d} = (5+4)\mathbf{i}+(-5+2)\mathbf{j} = 9\mathbf{i}-3\mathbf{j}$

(g) $2\mathbf{c}+5\mathbf{d} = (2+10)\mathbf{i}+(-2+5)\mathbf{j} = 12\mathbf{i}+3\mathbf{j}$

(h) $2\mathbf{c}-\mathbf{d} = (2-2)\mathbf{i}+(-2-1)\mathbf{j} = -3\mathbf{j}$

(i) $|\mathbf{d}-2\mathbf{c}| = |(2-2)\mathbf{i}+(1--2)\mathbf{j}| = |3\mathbf{j}| = 3$

(j) $|\mathbf{c}|+|\mathbf{d}| = \sqrt{1^2+1^2}+\sqrt{2^2+1^2} = \sqrt{2}+\sqrt{5} \approx 3.65$

(k) $|\mathbf{c}+\mathbf{d}| = |(1+2)\mathbf{i}+(-1+1)\mathbf{j}| = |3\mathbf{i}| = 3$

(l) $|\mathbf{c}-\mathbf{d}| = |(1-2)\mathbf{i}+(-1-1)\mathbf{j}| = |-1\mathbf{i}-2\mathbf{j}| = \sqrt{5} \approx 2.24$

14. (a) $\mathbf{a}+\mathbf{b} = \langle 5+2, 4+-3\rangle = \langle 7,1\rangle$

(b) $\mathbf{a}+-\mathbf{b} = \langle 5-2, 4--3\rangle = \langle 3,7\rangle$

(c) $2\mathbf{a} = 2\langle 5,4\rangle = \langle 10,8\rangle$

(d) $3\mathbf{a}+\mathbf{b} = \langle 3\times5+2, 3\times4+-3\rangle = \langle 17,9\rangle$

(e) $2\mathbf{b}-\mathbf{a} = \langle 4-5, -6-4\rangle = \langle -1,-10\rangle$

(f) $|\mathbf{a}| = \sqrt{5^2+4^2} = \sqrt{41} \approx 6.40$

(g) $|\mathbf{a}+\mathbf{b}| = \sqrt{7^2+1^2} = \sqrt{50} = 5\sqrt{2} \approx 7.07$

(h) $|\mathbf{a}|+|\mathbf{b}| = \sqrt{41}+\sqrt{2^2+3^2} = \sqrt{41}+\sqrt{13} \approx 10.01$

15. (a) $\begin{pmatrix}3\\4\end{pmatrix}+\begin{pmatrix}-1\\0\end{pmatrix} = \begin{pmatrix}2\\4\end{pmatrix}$

(b) $\begin{pmatrix}3\\4\end{pmatrix}-\begin{pmatrix}-1\\0\end{pmatrix} = \begin{pmatrix}4\\4\end{pmatrix}$

(c) $\begin{pmatrix}-1\\0\end{pmatrix}-\begin{pmatrix}3\\4\end{pmatrix} = \begin{pmatrix}-4\\-4\end{pmatrix}$

(d) $2\begin{pmatrix}3\\4\end{pmatrix}+\begin{pmatrix}-1\\0\end{pmatrix} = \begin{pmatrix}6\\8\end{pmatrix}+\begin{pmatrix}-1\\0\end{pmatrix} = \begin{pmatrix}5\\8\end{pmatrix}$

(e) $\begin{pmatrix}3\\4\end{pmatrix}+2\begin{pmatrix}-1\\0\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix}+\begin{pmatrix}-2\\0\end{pmatrix} = \begin{pmatrix}1\\4\end{pmatrix}$

(f) $\begin{pmatrix}3\\4\end{pmatrix}-2\begin{pmatrix}-1\\0\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix}-\begin{pmatrix}-2\\0\end{pmatrix} = \begin{pmatrix}5\\4\end{pmatrix}$

(g)
$$\left|\begin{pmatrix}3\\4\end{pmatrix}-2\begin{pmatrix}-1\\0\end{pmatrix}\right| = \left|\begin{pmatrix}5\\4\end{pmatrix}\right| = \sqrt{5^2+4^2} = \sqrt{41} \approx 6.40$$

(h)
$$\left|2\begin{pmatrix}-1\\0\end{pmatrix}-\begin{pmatrix}3\\4\end{pmatrix}\right| = \left|\begin{pmatrix}-5\\-4\end{pmatrix}\right| = \sqrt{5^2+4^2} = \sqrt{41} \approx 6.40$$

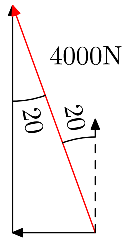
16. (a) $\left|\begin{pmatrix}2\\7\end{pmatrix}\right| = \sqrt{2^2+7^2} = \sqrt{53}$

(b) $\left|\begin{pmatrix}-2\\3\end{pmatrix}\right| = \sqrt{2^2+3^2} = \sqrt{13}$

(c) $\left|2\begin{pmatrix}2\\7\end{pmatrix}\right| = \sqrt{4^2+14^2} = \sqrt{212} = 2\sqrt{53}$

(d) $\left|\begin{pmatrix} 2 \\ 7 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix}\right| = \left|\begin{pmatrix} 0 \\ 10 \end{pmatrix}\right| = 10$

(e) $$\begin{aligned}\left|\begin{pmatrix} 2 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix}\right| &= \left|\begin{pmatrix} 4 \\ 4 \end{pmatrix}\right| \\ &= \sqrt{4^2+4^2} \\ &= \sqrt{32} \\ &= 4\sqrt{2}\end{aligned}$$

17. 

lift$= 4000\cos 20° = 3759$N
drag$= 4000\sin 20° = 1368$N

18. $(12\cos 50°\mathbf{i} + 12\sin 50°\mathbf{j}) + 10\mathbf{i} \approx (17.7\mathbf{i} + 9.2\mathbf{j})$N

19. $(-12\cos 50°\mathbf{i} + 12\sin 50°\mathbf{j}) + 10\mathbf{i} \approx (2.3\mathbf{i} + 9.2\mathbf{j})$N

20. $$\begin{aligned}\begin{pmatrix} -8\sin 40° \\ 8\cos 40° \end{pmatrix} + \begin{pmatrix} 5\cos 30° \\ 5\sin 30° \end{pmatrix} + \begin{pmatrix} 10 \\ 0 \end{pmatrix} &= \begin{pmatrix} -8\sin 40° + 5\cos 30° + 10 \\ 8\cos 40° + 5\sin 30° \end{pmatrix} \\ &\approx 9.2\mathbf{i} + 8.6\mathbf{j}\text{N}\end{aligned}$$

21. $$\begin{aligned}\begin{pmatrix} 0 + 10\cos 30° - 8\sin 20° \\ 6 + 10\sin 30° - 8\cos 20° \end{pmatrix} \\ = 5.9\mathbf{i} + 3.5\mathbf{j}\text{m/s}\end{aligned}$$

22. $$\begin{aligned} & 0\mathbf{i} + 5\mathbf{j} \\ + \quad & 10\cos 30°\mathbf{i} + 10\sin 30°\mathbf{j} \\ + \quad & 4\mathbf{i} + 0\mathbf{j} \\ + \quad & 7\cos 60°\mathbf{i} - 7\sin 60°\mathbf{j} \\ \approx \quad & (16.2\mathbf{i} + 3.9\mathbf{j})\text{N}\end{aligned}$$

23. $$\begin{aligned} & -10\sin 40°\mathbf{i} + 10\cos 40°\mathbf{j} \\ + \quad & 10\cos 30°\mathbf{i} + 10\sin 30°\mathbf{j} \\ + \quad & 10\cos 10°\mathbf{i} - 10\sin 10°\mathbf{j} \\ + \quad & -10\sin 10°\mathbf{i} - 10\cos 10°\mathbf{j} \\ \approx \quad & (10.3\mathbf{i} + 1.1\mathbf{j})\text{N}\end{aligned}$$

24. $$\begin{aligned}\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 &= (2+4+2)\mathbf{i} + (3+3-4)\mathbf{j} \\ &= (8\mathbf{i} + 2\mathbf{j})\text{N} \\ |\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3| &= |8\mathbf{i} + 2\mathbf{j}| \\ &= \sqrt{8^2+2^2} \\ &= 2\sqrt{17}\text{N}\end{aligned}$$

25. $$\begin{aligned}(\mathbf{a}+\mathbf{b}) + (\mathbf{a}-\mathbf{b}) &= (3\mathbf{i}+\mathbf{j}) + (\mathbf{i} - 7\mathbf{j}) \\ 2\mathbf{a} &= 4\mathbf{i} - 6\mathbf{j} \\ \mathbf{a} &= 2\mathbf{i} - 3\mathbf{j}\end{aligned}$$
$$\begin{aligned}(\mathbf{a}+\mathbf{b}) - (\mathbf{a}-\mathbf{b}) &= (3\mathbf{i}+\mathbf{j}) - (\mathbf{i} - 7\mathbf{j}) \\ 2\mathbf{b} &= 2\mathbf{i} + 8\mathbf{j} \\ \mathbf{b} &= \mathbf{i} + 4\mathbf{j}\end{aligned}$$

26. $$\begin{aligned}2(2\mathbf{c}+\mathbf{d}) - 2(\mathbf{c}+\mathbf{d}) &= 2(-\mathbf{i}+6\mathbf{j}) - (2\mathbf{i} - 10\mathbf{j}) \\ 2\mathbf{c} &= -4\mathbf{i} + 22\mathbf{j} \\ \mathbf{c} &= -2\mathbf{i} + 11\mathbf{j}\end{aligned}$$
$$\begin{aligned}(2\mathbf{c}+\mathbf{d}) - 2(\mathbf{c}+\mathbf{d}) &= (-\mathbf{i}+6\mathbf{j}) - (2\mathbf{i} - 10\mathbf{j}) \\ -\mathbf{d} &= -3\mathbf{i} + 16\mathbf{j} \\ \mathbf{d} &= 3\mathbf{i} - 16\mathbf{j}\end{aligned}$$

Exercise 4B

1.
- (a) $\mathbf{a} = 4\mathbf{i} + 3\mathbf{j}$
  (b) $2\mathbf{a} = 8\mathbf{i} + 6\mathbf{j}$
  (c) $\frac{\mathbf{a}}{|\mathbf{a}|} = \frac{4\mathbf{i}+3\mathbf{j}}{5} = 0.8\mathbf{i} + 0.6\mathbf{j}$
  (d) $2\frac{\mathbf{a}}{|\mathbf{a}|} = 2(0.8\mathbf{i} + 0.6\mathbf{j}) = 1.6\mathbf{i} + 1.2\mathbf{j}$
- (a) $\mathbf{b} = 4\mathbf{i} - 3\mathbf{j}$
  (b) $2\mathbf{b} = 8\mathbf{i} - 6\mathbf{j}$
  (c) $\frac{\mathbf{b}}{|\mathbf{b}|} = \frac{4\mathbf{i}-3\mathbf{j}}{5} = 0.8\mathbf{i} - 0.6\mathbf{j}$
  (d) $2\frac{\mathbf{b}}{|\mathbf{b}|} = 2(0.8\mathbf{i} - 0.6\mathbf{j}) = 1.6\mathbf{i} - 1.2\mathbf{j}$
- (a) $\mathbf{c} = 2\mathbf{i} + 2\mathbf{j}$
  (b) $2\mathbf{c} = 4\mathbf{i} + 4\mathbf{j}$
  (c) $\frac{\mathbf{c}}{|\mathbf{c}|} = \frac{2\mathbf{i}+2\mathbf{j}}{2\sqrt{2}} = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$
  (d) $2\frac{\mathbf{c}}{|\mathbf{c}|} = 2(\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}) = \sqrt{2}\mathbf{i} + \sqrt{2}\mathbf{j}$
- (a) $\mathbf{d} = 3\mathbf{i} - 2\mathbf{j}$
  (b) $2\mathbf{d} = 6\mathbf{i} - 4\mathbf{j}$
  (c) $\frac{\mathbf{d}}{|\mathbf{d}|} = \frac{3\mathbf{i}-2\mathbf{j}}{\sqrt{13}} = \frac{3}{\sqrt{13}}\mathbf{i} - \frac{2}{\sqrt{13}}\mathbf{j}$
  (d) $2\frac{\mathbf{d}}{|\mathbf{d}|} = 2(\frac{3}{\sqrt{13}}\mathbf{i} - \frac{2}{\sqrt{13}}\mathbf{j}) = \frac{6}{\sqrt{13}}\mathbf{i} - \frac{4}{\sqrt{13}}\mathbf{j}$

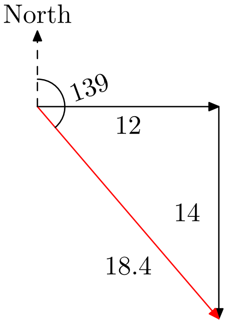
2. (a) $\frac{\mathbf{b}}{|\mathbf{b}|} = \frac{2\mathbf{i}+\mathbf{j}}{\sqrt{5}} = \frac{2}{\sqrt{5}}\mathbf{i} + \frac{1}{\sqrt{5}}\mathbf{j}$
   (b) $|\mathbf{a}|\frac{\mathbf{b}}{|\mathbf{b}|} = 5(\frac{2}{\sqrt{5}}\mathbf{i} + \frac{1}{\sqrt{5}}\mathbf{j}) = 2\sqrt{5}\mathbf{i} + \sqrt{5}\mathbf{j}$
   (c) $|\mathbf{c}|\frac{\mathbf{a}}{|\mathbf{a}|} = \sqrt{13}\frac{-3\mathbf{i}+4\mathbf{j}}{5} = -\frac{3\sqrt{13}}{5}\mathbf{i} + \frac{4\sqrt{13}}{5}\mathbf{j}$
   (d) $\mathbf{a}+\mathbf{b}+\mathbf{c} = 2\mathbf{i} + 3\mathbf{j}$
   $|\mathbf{a}+\mathbf{b}+\mathbf{c}| = \sqrt{13}$
   $|\mathbf{a}| = 5$
   $|\mathbf{a}|\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{|\mathbf{a}+\mathbf{b}+\mathbf{c}|} = 5\frac{2\mathbf{i}+3\mathbf{j}}{\sqrt{13}} = \frac{10}{\sqrt{13}}\mathbf{i} + \frac{15}{\sqrt{13}}\mathbf{j}$

3. (a) $\mathbf{a}$ and $\mathbf{d}$ are parallel since $\mathbf{a} = 2\mathbf{d}$.

(b) $\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}+\mathbf{e}$
$= (2+4+1+1+4)\mathbf{i}+(-4+2-8-2-2)\mathbf{j}$
$= 12\mathbf{i}-14\mathbf{j}$

(c) $|\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}+\mathbf{e}|$
$= \sqrt{12^2+14^2} = \sqrt{340}$
$= 2\sqrt{85}$

(d)



bearing$=90^\circ + \tan^{-1}\frac{14}{12} \approx 139^\circ$

4. • **a** is of magnitude 5 units and w is negative.

$$\begin{aligned}|\mathbf{a}| &= 5\\ |w\mathbf{i}+3\mathbf{j}| &= 5\\ \sqrt{w^2+3^2} &= 5\\ w^2+9 &= 25\\ w^2 &= 16\\ w &= -4\end{aligned}$$

• **b** is parallel to **a**

$$\begin{aligned}\mathbf{b} &= k\mathbf{a}\\ -\mathbf{i}+x\mathbf{j} &= k(w\mathbf{i}+3\mathbf{j})\\ -\mathbf{i}+x\mathbf{j} &= k(-4\mathbf{i}+3\mathbf{j})\\ -\mathbf{i}+x\mathbf{j} &= -4k\mathbf{i}+3k\mathbf{j}\\ (-1+4k)\mathbf{i} &= (3k-x)\mathbf{j}\\ -1+4k &= 0\\ k &= \frac{1}{4}\\ 3k-x &= 0\\ x &= \frac{3}{4}\end{aligned}$$

• **c** is a unit vector

$$\begin{aligned}|\mathbf{c}| &= 1\\ |0.5\mathbf{i}+y\mathbf{j}| &= 1\\ \sqrt{0.5^2+y^2} &= 1\\ 0.25+y^2 &= 1\\ y^2 &= \frac{3}{4}\\ y &= \pm\frac{\sqrt{3}}{2}\end{aligned}$$

• the resultant of **a** and **d** has a magnitude of 13 units

$$\begin{aligned}|\mathbf{a}+\mathbf{d}| &= 13\\ |(w-1)\mathbf{i}+(3-z)\mathbf{j}| &= 13\\ |(-4-1)\mathbf{i}+(3-z)\mathbf{j}| &= 13\\ \sqrt{5^2+(3-z)^2} &= 13\\ 25+(9-6z+z^2) &= 169\\ z^2-6z+9+25-169 &= 0\\ z^2-6z-135 &= 0\\ (z-15)(z+9) &= 0\\ z &= 15\\ \text{or } z &= -9\end{aligned}$$

$w=-4$; $x=\frac{3}{4}$; $y=\pm\frac{\sqrt{3}}{2}$; $z=15$ or 9.

5. • **p** is a unit vector and a is positive

$$\begin{aligned}|0.6\mathbf{i}-a\mathbf{j} &= 1\\ 0.6^2+a^2 &= 1^2\\ a &= 0.8\end{aligned}$$

• **q** is in the same direction as **p** and five times the magnitude.

$$\begin{aligned}\mathbf{q} &= 5\mathbf{p}\\ b\mathbf{i}+c\mathbf{i} &= 5(0.6\mathbf{i}-0.8\mathbf{j})\\ &= 3\mathbf{i}-4\mathbf{j}\\ b &= 3\\ c &= -4\end{aligned}$$

•
$$\begin{aligned}\mathbf{r}+2\mathbf{q} &= 11\mathbf{i}-20\mathbf{j}\\ (d\mathbf{i}+e\mathbf{j})+2(3\mathbf{i}-4\mathbf{j}) &= 11\mathbf{i}-20\mathbf{j}\\ (d+6)\mathbf{i}+(e-8)\mathbf{j} &= 11\mathbf{i}-20\mathbf{j}\\ d+6 &= 11\\ d &= 5\\ e-8 &= -20\\ e &= -12\end{aligned}$$

• **s** is in the same direction as **r** but equal in magnitude to **q**

$$\begin{aligned}\mathbf{s} &= |\mathbf{q}|\frac{\mathbf{r}}{|\mathbf{r}|}\\ f\mathbf{i}+g\mathbf{j} &= 5\frac{5\mathbf{i}-12\mathbf{j}}{\sqrt{5^2+12^2}}\\ &= \frac{25}{13}\mathbf{i}-\frac{60}{13}\mathbf{j}\\ f &= \frac{25}{13}\\ g &= -\frac{60}{13}\end{aligned}$$

$a=0.8$, $b=3$, $c=-4$, $d=5$, $e=-12$, $f=\frac{25}{13}$ and $g=-\frac{60}{13}$

6. $$\begin{aligned}\mathbf{R} &= \mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}\\ \mathbf{R} &= 7\cos 30^\circ\mathbf{i} + 7\sin 30^\circ\mathbf{j}\\ &\quad + 0\mathbf{i} + 6\mathbf{j}\\ &\quad + 10\cos 45^\circ\mathbf{i} + 10\sin 45^\circ\mathbf{j}\\ &\quad + 4\cos 145^\circ\mathbf{i} + 4\sin 145^\circ\mathbf{j}\\ \mathbf{R} &= 9.9\mathbf{i} + 18.9\mathbf{j}\\ |\mathbf{R}| &= \sqrt{9.9^2+18.9^2}\\ &= 21.3\\ \therefore\quad \mathbf{e} &= -9.9\mathbf{i} - 18.9\mathbf{j}\end{aligned}$$

7. $P = |(-6\mathbf{i}+5\mathbf{j})| = \sqrt{6^2+5^2} \approx 7.8$
   $\theta = \tan^{-1}\frac{6}{5} \approx 50^\circ$

8. Horizontal components:
   $$P\sin\theta = 8\sin 50^\circ$$
   Vertical components:
   $$P\cos\theta = 8\cos 50^\circ + 5$$
   Dividing gives:
   $$\begin{aligned}\frac{P\sin\theta}{P\cos\theta} &= \frac{8\sin 50^\circ}{8\cos 50^\circ+5}\\ \tan\theta &= \frac{8\sin 50^\circ}{8\cos 50^\circ+5}\\ \theta &= \tan^{-1}\frac{8\sin 50^\circ}{8\cos 50^\circ+5}\\ &\approx 31^\circ\end{aligned}$$
   Substituting:
   $$\begin{aligned}P\sin\theta &= 8\sin 50^\circ\\ P &= \frac{8\sin 50^\circ}{\sin 31^\circ} \qquad \approx 11.9\end{aligned}$$

9. Horizontal components:
   $$P\sin\theta = 12 - 10\sin 40^\circ$$
   Vertical components:
   $$P\cos\theta = 10\cos 40^\circ$$
   Dividing gives:
   $$\begin{aligned}\frac{P\sin\theta}{P\cos\theta} &= \frac{12-10\sin 40^\circ}{10\cos 40^\circ}\\ \tan\theta &= \frac{12-10\sin 40^\circ}{10\cos 40^\circ}\\ \theta &= \tan^{-1}\frac{12-10\sin 40^\circ}{10\cos 40^\circ}\\ &\approx 36^\circ\end{aligned}$$
   Substituting:
   $$\begin{aligned}P\cos\theta &= 10\cos 40^\circ\\ P &= \frac{10\cos 40^\circ}{\cos 36^\circ} \qquad \approx 9.5\end{aligned}$$

10. $$\begin{aligned}-T_1\sin 30^\circ + T_2\sin 30^\circ &= 0\\ \therefore\quad T_1 &= T_2\\ T_1\cos 30^\circ + T_2\cos 30^\circ &= 100\\ T_1\cos 30^\circ &= 50\\ T_1 &= \frac{50}{\cos 30^\circ}\\ &= \frac{100}{\sqrt{3}}\\ \therefore\quad T_1 = T_2 &= \frac{100}{\sqrt{3}}\text{N}\end{aligned}$$

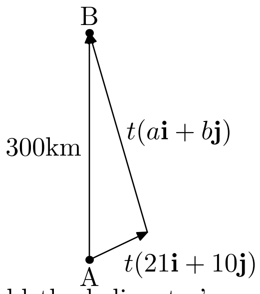
11. $$\begin{aligned}-T_1\sin 30^\circ + T_2\sin 30^\circ &= 0\\ \therefore\quad T_1 &= T_2\\ T_1\cos 60^\circ + T_2\cos 60^\circ &= 100\\ T_1\cos 60^\circ &= 50\\ T_1 &= \frac{50}{\cos 60^\circ}\\ &= 100\\ \therefore\quad T_1 = T_2 &= 100\text{N}\end{aligned}$$

12. First the horizontal components:
    $$\begin{aligned}-T_1\sin 30^\circ + T_2\sin 60^\circ &= 0\\ T_1\sin 30^\circ &= T_2\sin 60^\circ\\ \frac{1}{2}T_1 &= \frac{\sqrt{3}}{2}T_2\\ T_1 &= \sqrt{3}T_2\end{aligned}$$
    Now the vertical components:
    $$\begin{aligned}T_1\cos 30^\circ + T_2\cos 60^\circ &= 100\\ \frac{\sqrt{3}}{2}T_1 + \frac{1}{2}T_2 &= 100\\ \sqrt{3}T_1 + T_2 &= 200\end{aligned}$$
    Substituting:
    $$\begin{aligned}\sqrt{3}(\sqrt{3}T_2) + T_2 &= 200\\ 3T_2 + T_2 &= 200\\ 4T_2 &= 200\\ T_2 &= 50\text{N}\\ T_1 &= 50\sqrt{3}\text{N}\end{aligned}$$

13. Speed of A is $\sqrt{21^2+17^2} = \sqrt{730}$m/s.
    Speed of B is $\sqrt{26^2+2^2} = \sqrt{680}$m/s.
    Particle A is moving fastest.

14. Speed=$\sqrt{5^2+2^2} = \sqrt{29}$m/s.
    In one minute it will move $60\sqrt{29} \approx 323.1$m.

15. (a) When there is no wind blowing, the pilot flies due North with velocity vector $75\mathbf{j}$m/s for $300000 \div 75 = 4000$ seconds=1hr 6min 40sec.

(b) 

We must add the helicopter's own velocity to the wind velocity to produce a resultant headed due North.
Easterly ($\mathbf{i}$) components:

$$\begin{aligned}21+a&=0\\a&=-21\end{aligned}$$

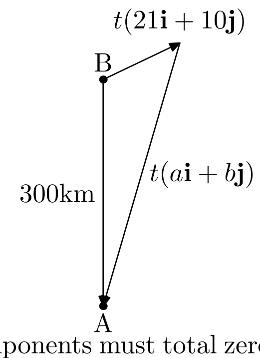
Now find the northerly ($\mathbf{j}$) component to give the correct speed:

$$\begin{aligned}\sqrt{a^2+b^2}&=75\\21^2+b^2&=75^2\\b&=\pm\sqrt{75^2-21^2}\\&=\pm72\end{aligned}$$

We know we're heading north, so we disregard the negative solution and conclude $b=72$.
To calculate time, we use the total northerly component (i.e. wind plus plane):

$$\begin{aligned}10t+bt&=300000\\82t&=300000\\t&=\frac{300000}{82}\\&\approx 3659\text{s}\\&\approx 61\text{min}\end{aligned}$$

The velocity vector is $(-21\mathbf{i}+72\mathbf{j})$m/s and the trip will take about one hour and one minute.

16. 

Easterly components must total zero, so as in the previous question $a=-21$.

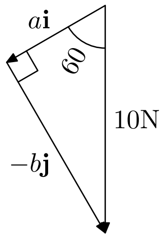
Calculation of the northerly component is the same as in the previous question, but this time we are heading southwards, so we reject the *positive* solution and conclude $b=-72$.

We now have a total southwards speed of $72-10=62$m/s so the time is

$$\begin{aligned}t&=\frac{300000}{62}\\&\approx 4839\text{s}\\&\approx 81\text{min}\end{aligned}$$

The velocity vector is $(-21\mathbf{i}-72\mathbf{j})$m/s and the trip will take about 81 minutes.

17. No working is required for this question. Refer to the answers in Sadler.

18. 

$a=10\cos 60°=5\text{N}$, $b=-10\sin 60°=-5\sqrt{3}\text{N}$.
The weight is $(5\mathbf{i}-5\sqrt{3}\mathbf{j})$N.

19. (a)
$$\begin{aligned}x(2\mathbf{i}+3\mathbf{j})+y(\mathbf{i}-\mathbf{j})&=3\mathbf{i}+2\mathbf{j}\\(2x+y)\mathbf{i}+(3x-y)\mathbf{j}&=3\mathbf{i}+2\mathbf{j}\\2x+y&=3\\3x-y&=2\end{aligned}$$
solving simultaneously:
$$\begin{aligned}x&=1\\y&=1\\3\mathbf{i}+2\mathbf{j}&=\mathbf{a}+\mathbf{b}\end{aligned}$$

(b)
$$\begin{aligned}(2x+y)\mathbf{i}+(3x-y)\mathbf{j}&=5\mathbf{i}+5\mathbf{j}\\2x+y&=5\\3x-y&=5\\x&=2\\y&=1\\5\mathbf{i}+5\mathbf{j}&=2\mathbf{a}+\mathbf{b}\end{aligned}$$

(c)
$$\begin{aligned}(2x+y)\mathbf{i}+(3x-y)\mathbf{j}&=\mathbf{i}+9\mathbf{j}\\2x+y&=1\\3x-y&=9\\x&=2\\y&=-3\\\mathbf{i}+9\mathbf{j}&=2\mathbf{a}-3\mathbf{b}\end{aligned}$$

(d)
$$\begin{aligned}(2x+y)\mathbf{i}+(3x-y)\mathbf{j}&=4\mathbf{i}+7\mathbf{j}\\2x+y&=4\\3x-y&=7\\x&=\frac{11}{5}\\y&=-\frac{2}{5}\\4\mathbf{i}+7\mathbf{j}&=\frac{11}{5}\mathbf{a}-\frac{2}{5}\mathbf{b}\end{aligned}$$

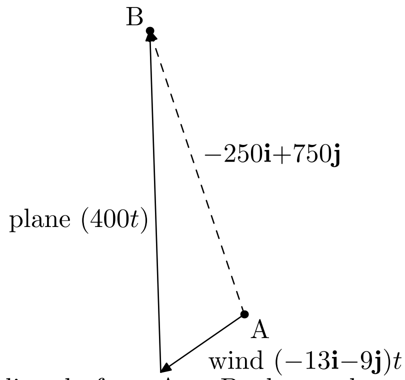
(e) $(2x+y)\mathbf{i}+(3x-y)\mathbf{j}=3\mathbf{i}-\mathbf{j}$

$$2x+y=3$$
$$3x-y=-1$$
$$x=\frac{2}{5}$$
$$y=\frac{11}{5}$$
$$3\mathbf{i}-\mathbf{j}=\frac{2}{5}\mathbf{a}+\frac{11}{5}\mathbf{b}$$

(f) $(2x+y)\mathbf{i}+(3x-y)\mathbf{j}=3\mathbf{i}+7\mathbf{j}$

$$2x+y=3$$
$$3x-y=7$$
$$x=2$$
$$y=-1$$
$$3\mathbf{i}+7\mathbf{j}=2\mathbf{a}-\mathbf{b}$$

20.



To fly directly from A to B, the resultant of the plane's velocity and the wind must be in the same direction as $\overrightarrow{AB}$. That is,

$$(a\mathbf{i}+b\mathbf{j})+(-13\mathbf{i}-9\mathbf{j})=(a-13)\mathbf{i}+(b-9)\mathbf{j}$$

in the same direction as

$$-250\mathbf{i}+750\mathbf{j}$$

Let θ represent this direction (an angle measured from the positive $\mathbf{i}$ direction) then

$$\tan\theta=\frac{750}{-250}=-3$$

and

$$\tan\theta=\frac{b-9}{a-13}$$

hence

$$\frac{b-9}{a-13}=-3$$
$$b-9=-3(a-13)$$
$$=-3a+39$$
$$b=48-3a$$

Now consider speed

$$a^2+b^2=400^2$$

and substitute for b:

$$a^2+(48-3a)^2=160\,000$$
$$a^2+2304-288a+9a^2=160\,000$$
$$10a^2-288a-157696=0$$

$a=-112$ or $a=140.8$

$b=48-3(-112)$ $b=48-3(140.8)$

$=384$ $=-374.4$

It should be clear that the first of these solutions takes us in the correct direction to go from A to B. The pilot should set a vector of $(-112\mathbf{i}+384\mathbf{j})$km/h for the trip from A to B.

For the return trip the same calculations apply (since $\tan(\theta+180°)=\tan\theta$) so we will get the same solutions for a and b, but here we will reject the first and accept the second.

The pilot should set a vector of $(140.8\mathbf{i}-374.4\mathbf{j})$km/h for the return trip from B to A.

Exercise 4C

1. (a) $\overrightarrow{OA}=2\mathbf{i}+5\mathbf{j}$

   (b) $\overrightarrow{OB}=-3\mathbf{i}+6\mathbf{j}$

   (c) $\overrightarrow{OC}=0\mathbf{i}-5\mathbf{j}$

   (d) $\overrightarrow{OD}=3\mathbf{i}+8\mathbf{j}$

2. (a) $\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}$
   $=-(3\mathbf{i}+\mathbf{j})+(2\mathbf{i}-\mathbf{j})$
   $=-\mathbf{i}-2\mathbf{j}$

   (b) $\overrightarrow{BA}=-\overrightarrow{AB}$
   $=\mathbf{i}+2\mathbf{j}$

3. (a) $\overrightarrow{AB}=-\overrightarrow{OA}+\overrightarrow{OB}$
   $=-(-\mathbf{i}+4\mathbf{j})+(2\mathbf{i}-3\mathbf{j})$
   $=3\mathbf{i}-7\mathbf{j}$

   (b) $\overrightarrow{BC}=-\overrightarrow{OB}+\overrightarrow{OC}$
   $=-(2\mathbf{i}-3\mathbf{j})+(\mathbf{i}+5\mathbf{j})$
   $=-\mathbf{i}+8\mathbf{j}$

(c) $\overrightarrow{\mathrm{CA}} = -(\mathbf{i}+5\mathbf{j}) + (-\mathbf{i}+4\mathbf{j})$
$= -2\mathbf{i} - \mathbf{j}$

4. (a) $\overrightarrow{\mathrm{AB}} = -(\mathbf{i}+2\mathbf{j}) + (4\mathbf{i}-2\mathbf{j})$
$= 3\mathbf{i} - 4\mathbf{j}$

(b) $\overrightarrow{\mathrm{BC}} = -(4\mathbf{i}-2\mathbf{j}) + (-\mathbf{i}+11\mathbf{j})$
$= -5\mathbf{i} + 13\mathbf{j}$

(c) $\overrightarrow{\mathrm{CD}} = -(-\mathbf{i}+11\mathbf{j}) + (6\mathbf{i}-13\mathbf{j})$
$= 7\mathbf{i} - 24\mathbf{j}$

(d) $|\overrightarrow{\mathrm{CD}}|\dfrac{\overrightarrow{\mathrm{AB}}}{|\overrightarrow{\mathrm{AB}}|} = 25\dfrac{3\mathbf{i}-4\mathbf{j}}{5}$
$= 15\mathbf{i} - 20\mathbf{j}$

5. (a) $|\overrightarrow{\mathrm{OA}}| = |3\mathbf{i}+7\mathbf{j}|$
$= \sqrt{3^2+7^2}$
$= \sqrt{58}$

(b) $|\overrightarrow{\mathrm{OB}}| = |-2\mathbf{i}+\mathbf{j}|$
$= \sqrt{2^2+1^2}$
$= \sqrt{5}$

(c) $|\overrightarrow{\mathrm{AB}}| = |-(3\mathbf{i}+7\mathbf{j}) + (-2\mathbf{i}+\mathbf{j})|$
$= |-5\mathbf{i}-6\mathbf{j}|$
$= \sqrt{5^2+6^2}$
$= \sqrt{61}$

6. (a) $|\overrightarrow{\mathrm{AB}}| = |3\mathbf{i}-4\mathbf{j}| = 5$

(b) $|\overrightarrow{\mathrm{BA}}| = |-3\mathbf{i}+4\mathbf{j}| = 5$

(c) $|\overrightarrow{\mathrm{AC}}| = |\mathbf{i}+4\mathbf{j}| = \sqrt{17}$

(d) $|\overrightarrow{\mathrm{BC}}| = |-2\mathbf{i}+8\mathbf{j}| = \sqrt{68} = 2\sqrt{17}$

7. (a) $\mathsf{OA} = \sqrt{1^2+6^2} = \sqrt{37}$

(b) $\mathsf{OB} = \sqrt{5^2+3^2} = \sqrt{34}$

(c) $\mathsf{BA} = \sqrt{(5--1)^2+(3-6)^2} = \sqrt{45} = 3\sqrt{5}$

8. (a) $\overrightarrow{\mathrm{AB}} = (1-2)\mathbf{i} + (2--3)\mathbf{j}$
$= -\mathbf{i} + 5\mathbf{j}$

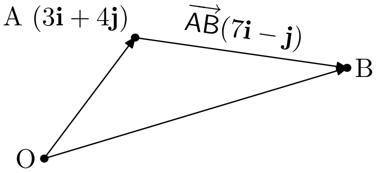
(b) $\overrightarrow{\mathrm{BC}} = (9-1)\mathbf{i} + (21-2)\mathbf{j}$
$= 8\mathbf{i} + 19\mathbf{j}$

(c) $\overrightarrow{\mathrm{CD}} = (6-9)\mathbf{i} + (-2-21)\mathbf{j}$
$= -3\mathbf{i} - 23\mathbf{j}$

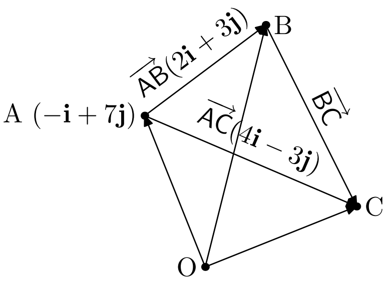
(d) $|\overrightarrow{\mathrm{AC}}| = |(9-2)\mathbf{i} + (21--3)\mathbf{j}|$
$= \sqrt{7^2+24^2}$
$= 25$

(e) $\overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}}$
$= \mathbf{i} + 2\mathbf{j}$

(f) $\overrightarrow{\mathrm{OA}} + 2\overrightarrow{\mathrm{AC}}$
$= (2\mathbf{i}-3\mathbf{j}) + 2\left((9-2)\mathbf{i} + (21--3)\mathbf{j}\right)$
$= 16\mathbf{i} + 45\mathbf{j}$

9. 

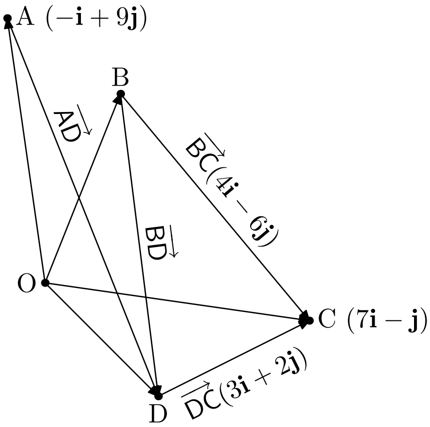
$\overrightarrow{\mathrm{OB}} = \overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{AB}}$
$= (3\mathbf{i}+4\mathbf{j}) + (7\mathbf{i}-\mathbf{j})$
$= 10\mathbf{i} + 3\mathbf{j}$

10. 

(a) $\overrightarrow{\mathrm{OB}} = \overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{AB}}$
$= (-\mathbf{i}+7\mathbf{j}) + (2\mathbf{i}+3\mathbf{j})$
$= \mathbf{i} + 10\mathbf{j}$

(b) $\overrightarrow{\mathrm{OC}} = \overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{AC}}$
$= (-\mathbf{i}+7\mathbf{j}) + (4\mathbf{i}-3\mathbf{j})$
$= 3\mathbf{i} + 4\mathbf{j}$

(c) $\overrightarrow{\mathrm{BC}} = -\overrightarrow{\mathrm{OB}} + \overrightarrow{\mathrm{OC}}$
$= -(\mathbf{i}+10\mathbf{j}) + (3\mathbf{i}+4\mathbf{j})$
$= 2\mathbf{i} - 6\mathbf{j}$

11. 

(a) $\overrightarrow{\mathrm{OB}} = \overrightarrow{\mathrm{OC}} + -\overrightarrow{\mathrm{BC}}$
$= (7\mathbf{i}-\mathbf{j}) - (4\mathbf{i}-6\mathbf{j})$
$= 3\mathbf{i} + 5\mathbf{j}$

(b) $\overrightarrow{\mathrm{OD}} = \overrightarrow{\mathrm{OC}} + -\overrightarrow{\mathrm{DC}}$
$= (7\mathbf{i}-\mathbf{j}) - (3\mathbf{i}+2\mathbf{j})$
$= 4\mathbf{i} - 3\mathbf{j}$

(c) $\overrightarrow{\mathrm{BD}} = \overrightarrow{\mathrm{BC}} - \overrightarrow{\mathrm{DC}}$
$= (4\mathbf{i}-6\mathbf{j}) - (3\mathbf{i}+2\mathbf{j})$
$= \mathbf{i} - 8\mathbf{j}$

(d) $|\overrightarrow{\mathrm{AD}}| = |-\overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{OD}}|$

$= |-(-\mathbf{i} + 9\mathbf{j}) + (4\mathbf{i} - 3\mathbf{j})|$

$= |5\mathbf{i} - 12\mathbf{j}|$

$= 13$

12. (a) $(2\mathbf{i} + 9\mathbf{j}) + (2\mathbf{i} - 5\mathbf{j}) = (4\mathbf{i} + 4\mathbf{j})\text{m}$
    (b) $(2\mathbf{i} + 9\mathbf{j}) + 2(2\mathbf{i} - 5\mathbf{j}) = (6\mathbf{i} - \mathbf{j})\text{m}$
    (c) $(2\mathbf{i} + 9\mathbf{j}) + 10(2\mathbf{i} - 5\mathbf{j}) = (22\mathbf{i} - 41\mathbf{j})\text{m}$
    (d) $|(2\mathbf{i} + 9\mathbf{j}) + 5(2\mathbf{i} - 5\mathbf{j})| = |12\mathbf{i} - 16\mathbf{j}| = 20\text{m}$

13. (a) $(5\mathbf{i} - 6\mathbf{j}) + 2(\mathbf{i} + 6\mathbf{j}) = (7\mathbf{i} + 6\mathbf{j})\text{m}$
    (b) $(5\mathbf{i} - 6\mathbf{j}) + 3(\mathbf{i} + 6\mathbf{j}) = (8\mathbf{i} + 12\mathbf{j})\text{m}$
    (c) $(5\mathbf{i} - 6\mathbf{j}) + 7(\mathbf{i} + 6\mathbf{j}) = (12\mathbf{i} + 36\mathbf{j})\text{m}$
    (d) $|(5\mathbf{i} - 6\mathbf{j}) + 5(\mathbf{i} + 6\mathbf{j})| = |10\mathbf{i} + 24\mathbf{j}| = 26\text{m}$
    (e)

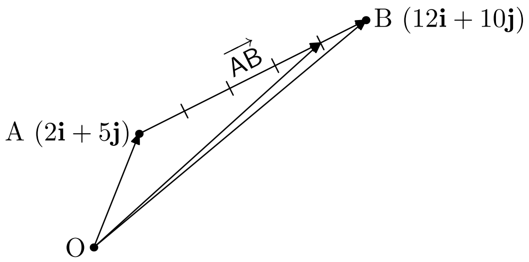
$$\begin{aligned}
|(5\mathbf{i} - 6\mathbf{j}) + t(\mathbf{i} + 6\mathbf{j})| &= 50 \\
|(5 + t)\mathbf{i} + (-6 + 6t)\mathbf{j}| &= 50 \\
\sqrt{(5 + t)^2 + (-6 + 6t)^2} &= 50 \\
(5 + t)^2 + (-6 + 6t)^2 &= 2500 \\
25 + 10t + t^2 + 36 - 72t + 36t^2 &= 2500 \\
37t^2 - 62t - 2439 &= 0 \\
t &= 9 \\
\text{or } t &= -\frac{271}{37}
\end{aligned}$$

The particle is 50m from the origin after 9 seconds.

14. If A, B and C are collinear, vectors $\overrightarrow{\mathrm{AB}}$, $\overrightarrow{\mathrm{AC}}$ and $\overrightarrow{\mathrm{BC}}$ will all be parallel; showing that any pair of these are scalar multiples of each other will demonstrate collinearity.
    $\overrightarrow{\mathrm{AB}} = -(3\mathbf{i} - \mathbf{j}) + (-\mathbf{i} + 15\mathbf{j}) = -4\mathbf{i} + 16\mathbf{j}$
    $\overrightarrow{\mathrm{AC}} = -(3\mathbf{i} - \mathbf{j}) + (9\mathbf{i} - 25\mathbf{j}) = 6\mathbf{i} - 24\mathbf{j}$
    $\overrightarrow{\mathrm{AC}} = -\frac{3}{2}\overrightarrow{\mathrm{AB}} \implies$ collinear.

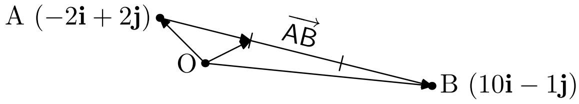
15. $\overrightarrow{\mathrm{DE}} = -(9\mathbf{i} - 7\mathbf{j}) + (-11\mathbf{i} + 8\mathbf{j}) = -20\mathbf{i} + 15\mathbf{j}$
    $\overrightarrow{\mathrm{DF}} = -(9\mathbf{i} - 7\mathbf{j}) + (25\mathbf{i} - 19\mathbf{j}) = 16\mathbf{i} - 12\mathbf{j}$
    $\overrightarrow{\mathrm{DE}} = -\frac{5}{4}\overrightarrow{\mathrm{DF}} \implies$ collinear.

16.



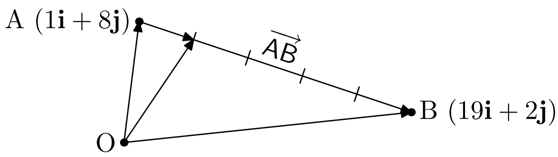
$$\begin{aligned}
\overrightarrow{\mathrm{OA}} + \frac{4}{5}\overrightarrow{\mathrm{AB}} &= (2\mathbf{i} + 5\mathbf{j}) + \frac{4}{5}\left(-(2\mathbf{i} + 5\mathbf{j}) + (12\mathbf{i} + 10\mathbf{j})\right) \\
&= (2\mathbf{i} + 5\mathbf{j}) + \frac{4}{5}(10\mathbf{i} + 5\mathbf{j}) \\
&= (2\mathbf{i} + 5\mathbf{j}) + (8\mathbf{i} + 4\mathbf{j}) \\
&= 10\mathbf{i} + 9\mathbf{j}
\end{aligned}$$

17.



$$\begin{aligned}
\overrightarrow{\mathrm{OA}} + \frac{1}{3}\overrightarrow{\mathrm{AB}} &= (-2\mathbf{i} + 2\mathbf{j}) + \frac{1}{3}\left(-(-2\mathbf{i} + 2\mathbf{j}) + (10\mathbf{i} - \mathbf{j})\right) \\
&= (-2\mathbf{i} + 2\mathbf{j}) + \frac{1}{3}(12\mathbf{i} - 3\mathbf{j}) \\
&= (-2\mathbf{i} + 2\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) \\
&= 2\mathbf{i} + \mathbf{j}
\end{aligned}$$

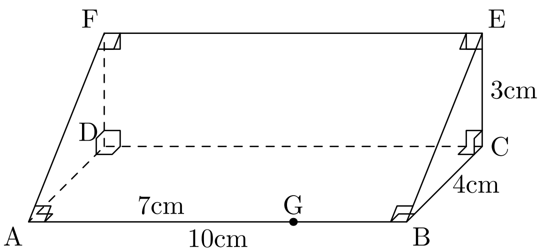
18.



$$\begin{aligned}
\overrightarrow{\mathrm{OA}} + \frac{1}{5}\overrightarrow{\mathrm{AB}} &= (\mathbf{i} + 8\mathbf{j}) + \frac{1}{5}\left(-(\mathbf{i} + 8\mathbf{j}) + (19\mathbf{i} + 2\mathbf{j})\right) \\
&= (\mathbf{i} + 8\mathbf{j}) + \frac{1}{5}(18\mathbf{i} - 6\mathbf{j}) \\
&= (\mathbf{i} + 8\mathbf{j}) + (3.6\mathbf{i} - 1.2\mathbf{j}) \\
&= 4.6\mathbf{i} + 6.8\mathbf{j}
\end{aligned}$$

Miscellaneous Exercise 4

1.



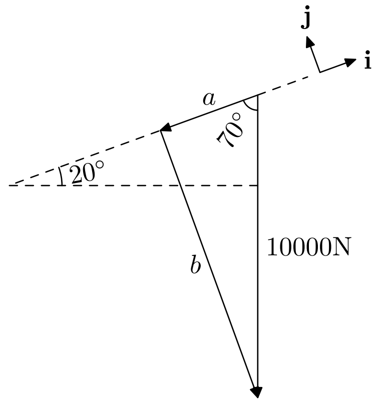
(a) $\tan\angle\mathrm{EBC} = \frac{3}{4}$
$\angle\mathrm{EBC} = \tan^{-1}\frac{3}{4} \approx 36.9°$

(b) To find ∠EGC we must first find the length GC.
Consider △BCG.
GB = 10 − AG = 3cm.
Using Pythagoras GC $= \sqrt{4^2 + 3^2} = 5$cm.
Now in △CGE,
$\tan\angle\mathrm{EGC} = \frac{3}{5}$
$\angle\mathrm{EGC} = \tan^{-1}\frac{3}{5} \approx 31.0°$

(c) To find ∠EAC we must first find the length AC.

Consider $\triangle$BCA.
Using Pythagoras AC $= \sqrt{4^2+10^2} = 2\sqrt{29}$cm.
Now in $\triangle$CAE,
$\tan\angle\text{EAC} = \frac{3}{2\sqrt{29}}$
$\angle\text{EAC} = \tan^{-1}\frac{3}{2\sqrt{29}} \approx 15.6°$

2.



(a) $a = 10000\cos 70° \approx 3400$
$b = 10000\sin 70° \approx 9400$
Weight=$(-3400\mathbf{i} - 9400\mathbf{j})$N.

(b) The resistance force the brakes must apply is equal and opposite the **i** component of the weight, that is 3 400 N.

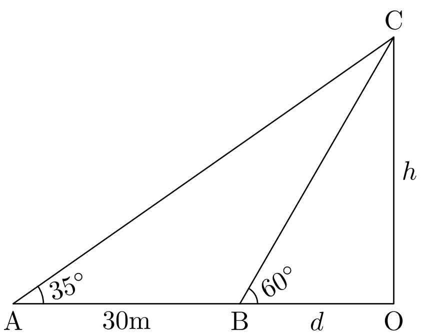
3.

$$\begin{aligned}\mathbf{c} &= \lambda\mathbf{a} + \mu\mathbf{b}\\ 2\mathbf{i}+\mathbf{j} &= \lambda(2\mathbf{i}+3\mathbf{j}) + \mu(3\mathbf{i}-4\mathbf{j})\\ 2\mathbf{i}+\mathbf{j} &= 2\lambda\mathbf{i}+3\lambda\mathbf{j}+3\mu\mathbf{i}-4\mu\mathbf{j}\\ (2-2\lambda-3\mu)\mathbf{i} &= (-1+3\lambda-4\mu)\mathbf{j}\end{aligned}$$

Since **i** and uj are not parallel, LHS and RHS must evaluate to the zero vector:

$$\begin{aligned}2\lambda+3\mu &= 2 && ①\\ 3\lambda-4\mu &= 1 && ②\\ 17\mu &= 4 && (3\times ① - 2\times ②)\\ \mu &= \frac{4}{17}\\ 3\lambda - \frac{16}{17} &= 1 && (\text{subst. } \mu \text{ into } ②)\\ 3\lambda &= \frac{33}{17}\\ \lambda &= \frac{11}{17}\end{aligned}$$

4.



First consider $\triangle$BCO:

$$\begin{aligned}\tan 60° &= \frac{h}{d}\\ \sqrt{3} &= \frac{h}{d}\\ d &= \frac{h}{\sqrt{3}} && \approx 0.577h\end{aligned}$$

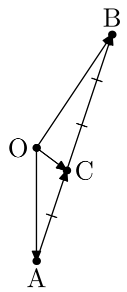
Now consider $\triangle$ACO:

$$\begin{aligned}\tan 35° &= \frac{h}{d+30}\\ d+30 &= \frac{h}{\tan 35°}\\ d &= \frac{h}{\tan 35°} - 30\\ &\approx 1.428h - 30\end{aligned}$$

combining these two results . . .

$$\begin{aligned}\therefore\ 1.428h - 30 &= 0.577h\\ (1.428-0.577)h &= 30\\ 0.851h &= 30\\ h &= \frac{30}{0.851}\\ &\approx 35\text{m}\end{aligned}$$

5.



$$\begin{aligned}\overrightarrow{AC} &= \frac{2}{3}\overrightarrow{CB}\\ \overrightarrow{OC} - \overrightarrow{OA} &= \frac{2}{3}(\overrightarrow{OB} - \overrightarrow{OC})\\ (4\mathbf{i}-3\mathbf{j}) - (a\mathbf{i}-15\mathbf{j}) &= \frac{2}{3}\left((10\mathbf{i}+b\mathbf{j}) - (4\mathbf{i}-3\mathbf{j})\right)\\ (4-a)\mathbf{i} + 12\mathbf{j} &= \frac{2}{3}\left(6\mathbf{i} + (b+3)\mathbf{j}\right)\\ &= 4\mathbf{i} + \frac{2(b+3)}{3}\mathbf{j}\end{aligned}$$

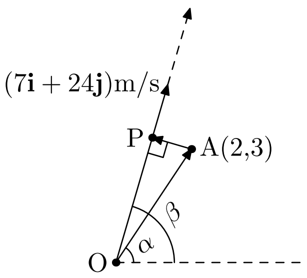
i components:

$$\begin{aligned}4-a &= 4\\ a &= 0\end{aligned}$$

j components:

$$\begin{aligned}12 &= \frac{2(b+3)}{3}\\ 18 &= b+3\\ b &= 15\end{aligned}$$

6. (a) The ball speed is $|7\mathbf{i} + 24\mathbf{j}| = \sqrt{7^2 + 24^2} = 25$m/s.
The time the ball takes to reach the boundary is $t = \frac{60}{25} = 2.4$s.

(b) 

Let P be the point of closest approach.

$$\begin{aligned}
\tan\alpha &= \frac{3}{2}\\
\alpha &= 56.3^\circ\\
\tan\beta &= \frac{24}{7}\\
\beta &= 73.7^\circ\\
\angle POA &= \beta - \alpha\\
&= 17.4^\circ\\
OA &= \sqrt{2^2 + 3^2}\\
&= \sqrt{13}\\
\sin 17.4^\circ &= \frac{AP}{OA}\\
AP &= OA\sin 17.4\\
&= \sqrt{13}\sin 17.4\\
&= 1.08\text{m}
\end{aligned}$$