Chapter 5

Exercise 5A

1. minor arc $AB = \frac{50}{360} \times 2\pi \times 12.4 \approx 10.8\text{cm}$
2. major arc $AB = \frac{235}{360} \times 2\pi \times 14.7 \approx 60.3\text{cm}$
3. minor arc $AB = \frac{360-290}{360} \times 2\pi \times 6.7 \approx 8.2\text{cm}$
4. major arc $AB = \frac{360-120}{360} \times 2\pi \times 8 = \frac{2}{3} \times 16\pi = \frac{32\pi}{3}\text{cm}$
5. minor arc $AB = \frac{150}{360} \times 2\pi \times 10 = \frac{25\pi}{3}\text{cm}$
6. major arc $AB = \frac{280}{360} \times 2\pi \times 6 = \frac{28\pi}{3}\text{cm}$
7. minor sector$= \frac{60}{360}\pi \times 12^2 = 24\pi\text{cm}^2$
8. minor sector$= \frac{110}{360}\pi \times 6^2 = 11\pi\text{cm}^2$
9. major sector$= \frac{360-120}{360}\pi \times 8^2 = \frac{128\pi}{3}\text{cm}^2$
10. minor sector$= \frac{360-205}{360}\pi \times 15.4^2 \approx 321\text{cm}^2$
11. First, from arc length l to angle θ

$$\begin{aligned} l &= \frac{\theta}{360} 2\pi r \\ \theta &= \frac{360l}{2\pi r} \end{aligned}$$

Then from angle θ to sector area a

$$\begin{aligned} a &= \frac{\theta}{360}\pi r^2 \\ &= \frac{\frac{360l}{2\pi r}}{360}\pi r^2 \\ &= \frac{360\pi l r^2}{720\pi r} \\ &= \frac{lr}{2} \end{aligned}$$

So for question 11
minor sector$= \frac{12.3\times17.6}{2} \approx 108\text{cm}^2$

12. major sector$= \frac{40\times10}{2} = 200\text{cm}^2$
13.

$$\begin{aligned} \text{minor segment} &= \text{minor sector} \quad - \text{triangle} \\ &= \frac{100}{360}\pi \times 15^2 \quad - \frac{15^2 \times \sin 100}{2} \\ &\approx 86\text{cm}^2 \end{aligned}$$

14.

$$\begin{aligned} \theta &= \frac{360l}{2\pi r} \\ &= \frac{288}{\pi} \\ \text{minor segment} &= \frac{\theta}{360}\pi r^2 - \frac{r^2 \sin\theta}{2} \\ &= \frac{288}{360} \times 10^2 - \frac{10^2 \sin\left(\frac{288}{\pi}\right)}{2} \\ &\approx 80.0 - 50.0 \\ &\approx 30\text{cm}^2 \end{aligned}$$

15.

$$\begin{aligned} \theta &= \frac{90}{\pi \times 10^2} \times 360 \\ &= \frac{324}{\pi} \\ \text{minor segment} &= 90 - \frac{10^2 \sin\frac{324}{\pi}}{2} \\ &\approx 41\text{cm}^2 \end{aligned}$$

16.

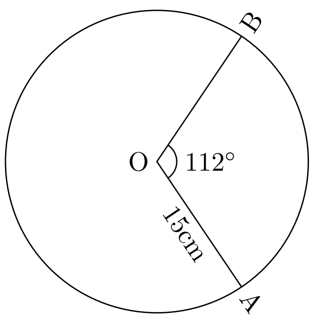
$$\begin{aligned} \text{minor segment} &= \frac{60}{360}\pi \times 12^2 - \frac{1}{2}12^2 \sin 60^\circ \\ &= 24\pi - 72 \times \frac{\sqrt{3}}{2} \\ &= 24\pi - 36\sqrt{3} \\ &= 12(2\pi - 3\sqrt{3})\text{cm}^2 \end{aligned}$$

17.

$$\begin{aligned} \text{minor segment} &= \frac{135}{360}\pi \times 6^2 - \frac{1}{2}6^2 \sin 135^\circ \\ &= \frac{27\pi}{2} - 18 \times \frac{\sqrt{2}}{2} \\ &= \frac{27\pi}{2} - 9\sqrt{2} \\ &= 9\left(\frac{3\pi}{2} - \sqrt{2}\right)\text{cm}^2 \end{aligned}$$

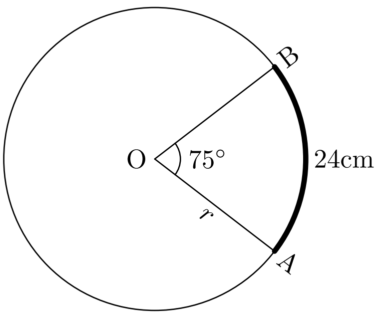
18.

$$\begin{aligned} \theta &= 360 - 210 \\ &= 150^\circ \\ \text{minor segment} &= \frac{150}{360}\pi \times 10^2 - \frac{1}{2}10^2 \sin(150)^\circ \\ &= \frac{125\pi}{3} - 50 \sin 150^\circ \\ &= \frac{125\pi}{3} - 25 \\ &= \frac{25}{3}(5\pi - 3)\text{cm}^2 \end{aligned}$$

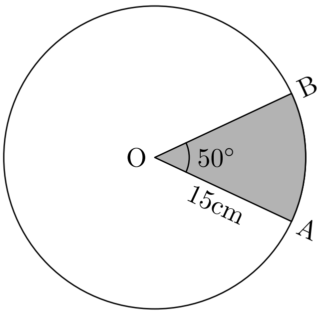
19. 

(a) minor arc$= \frac{112}{360} \times 2\pi \times 15 = \frac{28\pi}{3} \approx 29.3\text{cm}$

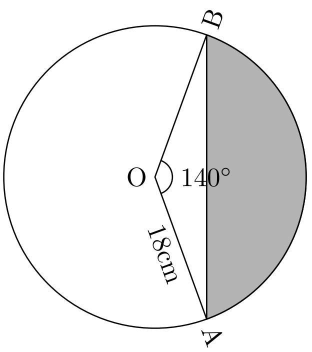
(b) major arc$= \frac{360-112}{360} \times 2\pi \times 15 = \frac{62\pi}{3} \approx 64.9\text{cm}$

20. 

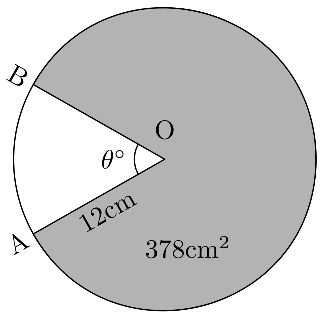
$$\frac{75}{360} \times 2\pi r = 24$$
$$r = \frac{24}{2\pi} \times \frac{360}{75}$$
$$= \frac{288}{5\pi}$$
$$\approx 18.3\text{cm}$$

21. 

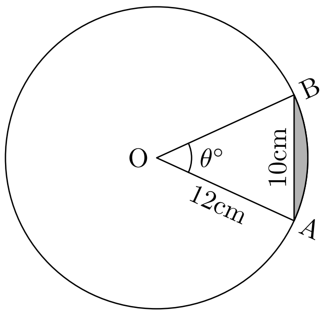
$$a = \frac{50}{360}\pi \times 15^2$$
$$= \frac{125\pi}{4}$$
$$\approx 98.2\text{cm}^2$$

22. 

$$\angle\text{OAB} = 20°$$
$$\implies \quad \text{OAB} = 180 - 2 \times 20$$
$$= 140°$$
$$a = \frac{140}{360}\pi \times 18^2 - \frac{1}{2}18^2 \sin 140°$$
$$\approx 292\text{cm}^2$$

23. 

$$378 = \frac{360 - \theta}{360}\pi \times 12^2$$
$$360 - \theta = \frac{378 \times 360}{144\pi}$$
$$\approx 300.8$$
$$\theta \approx 59°$$

24. 

$$\theta = \cos^{-1}\frac{12^2 + 12^2 - 10^2}{2 \times 12 \times 12}$$
$$\approx 49.2°$$
$$a = \frac{49.2}{360}\pi \times 12^2 - \frac{1}{2}12^2 \sin 49.2°$$
$$\approx 7.3\text{cm}^2$$

25. In half an hour the minute hand sweeps out 180° or half a circle. Its tip travels

$$d = \frac{1}{2} \times 2\pi \times 12$$
$$= 12\pi\text{cm}$$

In half an hour the hour hand sweeps out $\frac{1}{24}$ of a full circle. Its tip travels

$$d = \frac{1}{24} \times 2\pi \times 8$$
$$= \frac{2\pi}{3}\text{cm}$$

26. The ship is travelling through 3 degrees of latitude. $3° = 3 \times 60 = 180'$. The ship travels 180 nautical miles.
One nautical mile in kilometres is

$$d = \frac{\frac{1}{60}}{360} \times 2\pi \times 6350$$
$$\approx 1.85\text{km}$$

27. The circumference of the base of the cone is equal to the arc length of the sector:

$$\begin{aligned} 2\pi r &= \frac{240}{360} \times 2\pi \times 10 \\ r &= \frac{20}{3} \end{aligned}$$

The slant height of the cone is the radius of the sector:

$$\begin{aligned} h^2 + r^2 &= 10^2 \\ h &= \sqrt{10^2 - \left(\frac{20}{3}\right)^2} \\ &= \sqrt{100 - \frac{400}{9}} \\ &= 10\sqrt{1 - \frac{4}{9}} \\ &= 10\sqrt{\frac{5}{9}} \\ &= \frac{10\sqrt{5}}{3} \end{aligned}$$

Exercise 5B

1. $\theta = \frac{3}{1} = 3$ rads
2. $\theta = \frac{3}{2} = 1.5$ rads
3. $\theta = \frac{5}{1} = 5$
4. $\theta = \frac{5}{2} = 2.5$
5. $\theta = \frac{4}{1} = 4$
6. $\theta = \frac{8}{2} = 4$
7. $5° = 5 \times \frac{\pi}{180} = \frac{\pi}{36}$
8. $18° = 18 \times \frac{\pi}{180} = \frac{\pi}{10}$
9. $30° = 30 \times \frac{\pi}{180} = \frac{\pi}{6}$
10. $80° = 80 \times \frac{\pi}{180} = \frac{4\pi}{9}$
11. $144° = 144 \times \frac{\pi}{180} = \frac{4\pi}{5}$
12. $40° = 40 \times \frac{\pi}{180} = \frac{2\pi}{9}$
13. $145° = 145 \times \frac{\pi}{180} = \frac{29\pi}{36}$
14. $108° = 108 \times \frac{\pi}{180} = \frac{3\pi}{5}$
15. $165° = 165 \times \frac{\pi}{180} = \frac{11\pi}{12}$
16. $9° = 9 \times \frac{\pi}{180} = \frac{\pi}{20}$
17. $65° = 65 \times \frac{\pi}{180} = \frac{13\pi}{36}$
18. $110° = 110 \times \frac{\pi}{180} = \frac{11\pi}{18}$
19. $130° = 130 \times \frac{\pi}{180} = \frac{13\pi}{18}$
20. $126° = 126 \times \frac{\pi}{180} = \frac{7\pi}{10}$
21. $99° = 99 \times \frac{\pi}{180} = \frac{11\pi}{20}$
22. $155° = 155 \times \frac{\pi}{180} = \frac{31\pi}{36}$
23. $\frac{\pi}{6}$ rads $= \frac{\pi}{6} \times \frac{180}{\pi} = 30°$
24. $\frac{\pi}{12}$ rads $= \frac{\pi}{12} \times \frac{180}{\pi} = 15°$
25. $\frac{5\pi}{18}$ rads $= \frac{5\pi}{18} \times \frac{180}{\pi} = 50°$
26. $\frac{3\pi}{10}$ rads $= \frac{3\pi}{10} \times \frac{180}{\pi} = 54°$
27. $\frac{2\pi}{5}$ rads $= \frac{2\pi}{5} \times \frac{180}{\pi} = 72°$
28. $\frac{8\pi}{9}$ rads $= \frac{8\pi}{9} \times \frac{180}{\pi} = 160°$
29. π rads $= 180°$
30. $\frac{35\pi}{36}$ rads $= \frac{35\pi}{36} \times \frac{180}{\pi} = 175°$
31. $\frac{\pi}{2}$ rads $= \frac{\pi}{2} \times \frac{180}{\pi} = 90°$
32. $\frac{3\pi}{8}$ rads $= \frac{3\pi}{8} \times \frac{180}{\pi} = 67.5°$
33. $\frac{\pi}{3}$ rads $= \frac{\pi}{3} \times \frac{180}{\pi} = 60°$
34. $\frac{\pi}{5}$ rads $= \frac{\pi}{5} \times \frac{180}{\pi} = 36°$
35. $\frac{17\pi}{36}$ rads $= \frac{17\pi}{36} \times \frac{180}{\pi} = 85°$
36. $\frac{3\pi}{4}$ rads $= \frac{3\pi}{4} \times \frac{180}{\pi} = 135°$
37. $\frac{11\pi}{60}$ rads $= \frac{11\pi}{60} \times \frac{180}{\pi} = 33°$
38. $\frac{7\pi}{18}$ rads $= \frac{7\pi}{18} \times \frac{180}{\pi} = 70°$
39. $32° = 32 \times \frac{\pi}{180} \approx 0.56$
40. $63° = 63 \times \frac{\pi}{180} \approx 1.10$
41. $115° = 115 \times \frac{\pi}{180} \approx 2.01$
42. $170° = 170 \times \frac{\pi}{180} \approx 2.97$
43. $16° = 16 \times \frac{\pi}{180} \approx 0.28$
44. $84° = 84 \times \frac{\pi}{180} \approx 1.47$
45. $104° = 104 \times \frac{\pi}{180} \approx 1.82$
46. $26° = 26 \times \frac{\pi}{180} \approx 0.45$

47. $76° = 76 \times \frac{\pi}{180} \approx 1.33$

48. $51° = 51 \times \frac{\pi}{180} \approx 0.89$

49. $152° = 152 \times \frac{\pi}{180} \approx 2.65$

50. $158° = 158 \times \frac{\pi}{180} \approx 2.76$

51. $1.5^R = 1.5 \times \frac{180}{\pi} \approx 86°$

52. $2.3^R = 2.3 \times \frac{180}{\pi} \approx 132°$

53. $1.4^R = 1.4 \times \frac{180}{\pi} \approx 80°$

54. $0.6^R = 0.6 \times \frac{180}{\pi} \approx 34°$

55. $0.2^R = 0.2 \times \frac{180}{\pi} \approx 11°$

56. $0.32^R = 0.32 \times \frac{180}{\pi} \approx 18°$

57. $1.21^R = 1.21 \times \frac{180}{\pi} \approx 69°$

58. $3.1^R = 3.1 \times \frac{180}{\pi} \approx 178°$

For exact value problems, you should work towards knowing these in radians as well as degrees, so you don't have to first convert to degrees. This will come with time and effort. You should deliberately memorise the following table:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	undefined

59. $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

60. $\frac{5\pi}{6}$ makes an angle of $\frac{\pi}{6}$ with the x-axis and is in quadrant 2, so $\sin\frac{5\pi}{6} = \frac{1}{2}$

61. $\frac{3\pi}{4}$ makes an angle of $\frac{\pi}{4}$ with the x-axis and is in quadrant 2, so $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

62. $\sin\frac{\pi}{2} = 1$

63. $\frac{2\pi}{3}$ makes an angle of $\frac{\pi}{3}$ with the x-axis and is in quadrant 2, so $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

64. $\frac{3\pi}{4}$ makes an angle of $\frac{\pi}{4}$ with the x-axis and is in quadrant 2, so $\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$

65. $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

66. $\frac{2\pi}{3}$ makes an angle of $\frac{\pi}{3}$ with the x-axis and is in quadrant 2, so $\tan\frac{2\pi}{3} = -\sqrt{3}$

67. $\cos\frac{\pi}{2} = 0$

68. $\tan\frac{\pi}{2}$ is undefined

69. $\frac{2\pi}{3}$ makes an angle of $\frac{\pi}{3}$ with the x-axis and is in quadrant 2, so $\cos\frac{2\pi}{3} = -\frac{1}{2}$

70. $\frac{5\pi}{6}$ makes an angle of $\frac{\pi}{6}$ with the x-axis and is in quadrant 2, so $\tan\frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$

71. $\frac{5\pi}{6}$ makes an angle of $\frac{\pi}{6}$ with the x-axis and is in quadrant 2, so $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

72. π makes an angle of 0 with the x-axis and is in quadrant 2, so $\tan\pi = 0$

73. $\cos\frac{\pi}{3} = \frac{1}{2}$

74. π makes an angle of 0 with the x-axis and is in quadrant 2, so $\sin\pi = 0$

Questions 75–90 are single-step calculator exercises, so there's no point in reproducing the solutions here. Refer to the answers in Sadler.

91. (a) 3 revolutions/second$= 3 \times 2\pi = 6\pi$ radians/second.
    (b) 15 revolutions/minute$= \frac{15}{60} \times 2\pi = \frac{\pi}{2}$ radians/second.
    (c) 90 degrees/second$= \frac{\pi}{4}$ radians/second.

92. (a) 2π radians/minute $= 1$ revolution/minute
    (b) $\frac{3\pi}{4}$ radians/second $= \frac{3\pi}{4} \times 60 = 45\pi$ radians/minute $= \frac{45\pi}{2\pi} = 22.5$ revolutions/minute
    (c) $\frac{\pi}{3}$ radians/second $= \frac{\pi}{3} \times 60 = 20\pi$ radians/minute $= \frac{20\pi}{2\pi} = 10$ revolutions/minute

93.
$$\begin{aligned}\sin 1 &= \frac{6}{x}\\ x &= \frac{6}{\sin 1}\\ &\approx 7.1\end{aligned}$$

94.
$$\begin{aligned}\tan 1.2 &= \frac{8}{x}\\ x &= \frac{8}{\tan 1.2}\\ &\approx 3.1\end{aligned}$$

95. Let h be the perpendicular height in cm.
$$\begin{aligned}\sin 0.6 &= \frac{h}{20}\\ h &= 20\sin 0.6\\ &\approx 11.3\\ x &= \sqrt{h^2 + 6^2}\\ &\approx 12.8\end{aligned}$$

96.
$$\begin{aligned}\frac{x}{\sin 1.1} &= \frac{14}{\sin 1.8}\\ x &= \frac{14\sin 1.1}{\sin 1.8}\\ &= 12.8\end{aligned}$$

97.
$$\begin{aligned}\theta &= \pi - 0.64\\ &\approx 2.50\\ x &= \sqrt{7^2 + 10^2 - 2\times 7\times 10\cos 2.50}\\ &\approx 16.2\end{aligned}$$

98. $7.2^2 = 5.0^2 + 6.1^2 - 2 \times 5.0 \times 6.1 \cos x$

$$x = \cos^{-1}\left(\frac{5.0^2 + 6.1^2 - 7.2^2}{2 \times 5.0 \times 6.1}\right)$$
$$\approx 1.4$$

99. (a) $\frac{1}{4} \times 2\pi = \frac{\pi}{2}$

(b) $\frac{2}{3} \times 2\pi = \frac{4\pi}{3}$

(c) $\frac{5}{6} \times 2\pi = \frac{5\pi}{3}$

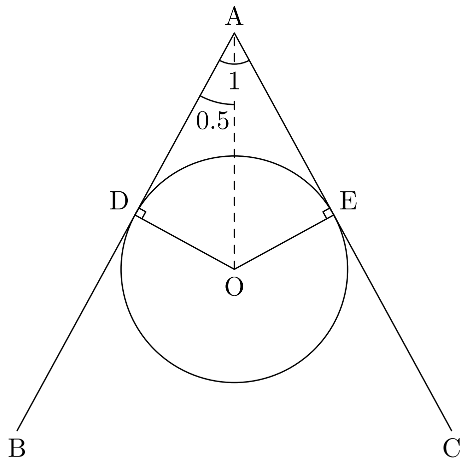
(d) $\frac{55}{60} \times 2\pi = \frac{11\pi}{6}$

100. (a) 50 grads$= 0.50 \times \frac{\pi}{2} = \frac{\pi}{4}$

(b) 75 grads$= 0.75 \times \frac{\pi}{2} = \frac{3\pi}{8}$

(c) 10 grads$= 0.10 \times \frac{\pi}{2} = \frac{\pi}{20}$

(d) 130 grads$= 0.10 \times \frac{\pi}{2} = \frac{13\pi}{20}$



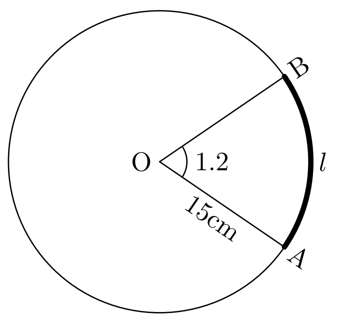
101. (a) Let d be the diameter of the pipe.

$$\frac{\text{DO}}{\text{DA}} = \tan 0.5$$
$$\text{DA} = \frac{\text{DO}}{\tan 0.5}$$
$$= \frac{d}{2\tan 0.5}$$
$$= \frac{d}{1.09}$$
$$= 0.915d$$

The scale along AB must be set up so that 1cm units are 0.915cm apart, starting with 0 at A.

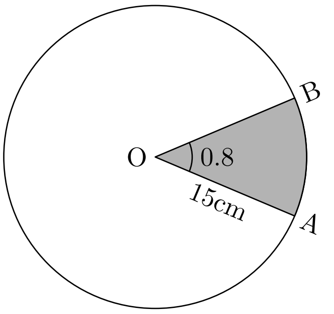
(b) If BAC $= \frac{\pi}{2}$ then $\triangle$ODA is isosceles so OD=AD or AD $= 0.5d$. This would be simpler to construct as 1cm units along AB would be exactly 0.5cm apart.

Exercise 5C

1. $l = r\theta = 5 \times 0.8 = 4\text{cm}$
2. $l = r\theta = 10 \times 2.5 = 25\text{cm}$
3. $l = r\theta = 7.8 \times (2\pi - 4.5) \approx 13.9\text{cm}$
4. $a = \frac{1}{2}r^2\theta$
$= \frac{1}{2} \times 4^2 \times 1$
$= 8\text{cm}^2$
5. $a = \frac{1}{2}r^2\theta$
$= \frac{1}{2} \times 6^2 \times 2.5$
$= 45\text{cm}^2$
6. $a = \frac{1}{2}r^2\theta$
$= \frac{1}{2} \times 10^2 \times (2\pi - 4)$
$= 114.2\text{cm}^2$
7. $a = \frac{1}{2}r^2(\theta - \sin\theta)$
$= \frac{1}{2} \times 59^2(1 - \sin 1)$
$= 275.9\text{cm}^2$
8. $a = \frac{1}{2}r^2(\theta - \sin\theta)$
$= \frac{1}{2} \times 5^2\left((2\pi - 3.5) - \sin(2\pi - 3.5)\right)$
$= 30.4\text{cm}^2$
9. $a = \frac{1}{2}r^2(\theta - \sin\theta)$
$= \frac{1}{2} \times 7.5^2(2.2 - \sin 2.2)$
$= 39.1\text{cm}^2$
10. 

$l = r\theta = 15 \times 1.2 = 18\text{cm}$

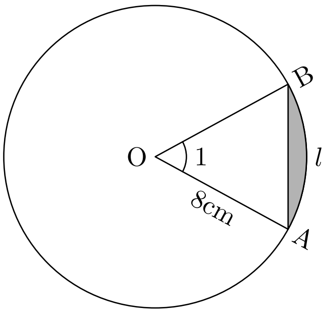
11.



(a) $a = \frac{1}{2}r^2\theta = \frac{1}{2} \times 15^2 \times 0.8 = 90\text{cm}^2$

(b) $a = \pi \times 15^2 - 90 \approx 616.9\text{cm}^2$

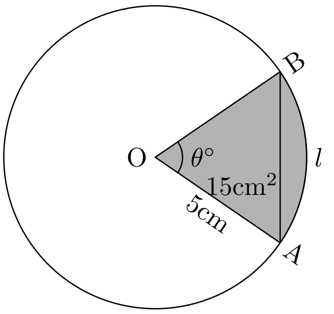
12.



(a) $l = r\theta = 8 \times 1 = 8\text{cm}$

(b)
$$\begin{aligned} a &= \frac{1}{2}r^2(\theta - \sin\theta) \\ &= \frac{1}{2} \times 8^2(1 - sin1) \\ &\approx 5.1\text{cm}^2 \end{aligned}$$

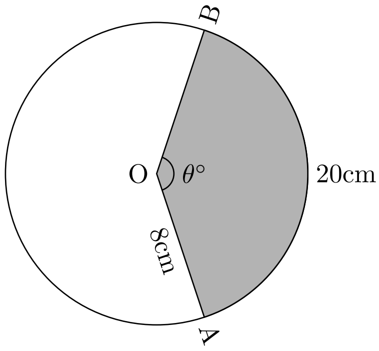
13.



(a)
$$\begin{aligned} \frac{1}{2}r^2\theta &= 15 \\ \frac{25}{2}\theta &= 15 \\ \theta &= \frac{6}{5} \\ l &= r\theta \\ &= 5 \times \frac{6}{5} \\ &= 6\text{cm} \end{aligned}$$

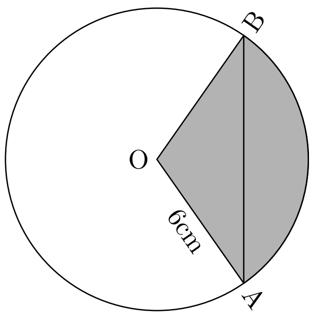
(b)
$$\begin{aligned} a &= \frac{1}{2}r^2(\theta - \sin\theta) \\ &= \frac{1}{2} \times 5^2(\frac{6}{5} - sin\frac{6}{5}) \\ &\approx 3.35\text{cm}^2 \end{aligned}$$

14.



$$\begin{aligned} \theta &= l/r \\ &= 20/8 \\ &= 2.5 \\ a &= \frac{1}{2}r^2\theta \\ &= \frac{1}{2} \times 8^2 \times 2.5 \\ &= 80\text{cm}^2 \end{aligned}$$

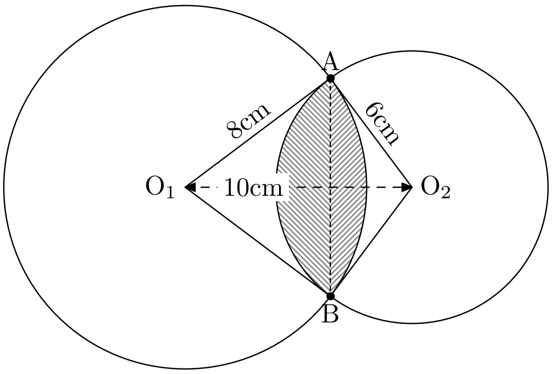
15.



$$\begin{aligned} \frac{1}{2}r^2\theta &= 9 \\ \frac{1}{2} \times 6^2\theta &= 9 \\ 18\theta &= 9 \\ \theta &= \frac{1}{2} \\ a &= \frac{1}{2}r^2(\theta - \sin\theta) \\ &= \frac{1}{2} \times 6^2\left(\frac{1}{2} - \sin\frac{1}{2}\right) \quad \approx 0.37\text{cm}^2 \end{aligned}$$

16.
$$\begin{aligned} a &= \text{segment BOC} - \text{segment AOD} \\ &= \frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta \\ &= \frac{\theta}{2}(R^2 - r^2) \\ &= \frac{1.5}{2}(12^2 - 6^2) \\ &= 81\text{cm}^2 \end{aligned}$$

17.
$$\begin{aligned} a &= \frac{\theta}{2}(R^2 - r^2) \\ &= \frac{1.5}{2}(9^2 - 5^2) \\ &= 42\text{cm}^2 \end{aligned}$$

18. 

Consider $\triangle O_1AO_2$.

$$\begin{aligned}\angle AO_1O_2 &= \cos^{-1}\frac{8^2+10^2-6^2}{2\times8\times10}\\ &\approx 0.644\\ \angle AO_1B &= 2\angle AO_1O_2\\ &\approx 1.287\\ \text{segment } AO_1B &= \frac{1}{2}\times8^2(1.287-\sin1.287)\\ &\approx 10.46\end{aligned}$$

Similarly

$$\begin{aligned}\angle AO_2O_1 &= \cos^{-1}\frac{6^2+10^2-8^2}{2\times6\times10}\\ &\approx 0.927\\ \angle AO_2B &= 2\angle AO_2O_1\\ &\approx 1.855\\ \text{segment } AO_2B &= \frac{1}{2}\times6^2(1.855-\sin1.855)\\ &\approx 16.10\\ \text{total area} &= 10.46+16.10\\ &= 26.57\text{cm}^2\end{aligned}$$

19. $$\begin{aligned}\text{area}_{\text{segment BOC}} &= \frac{1}{2}\times8^2\times0.8\\ &= 25.6\\ \text{area}_{\triangle\text{AOD}} &= \frac{1}{2}\times5^2\sin0.8\\ &\approx 8.97\\ \text{area}_{\text{ABCD}} &= 25.6-8.97\\ &= 16.60\text{cm}^2\end{aligned}$$

20. The AC and BC are perpendicular to AO and BO respectively, since a tangent is perpendicular to a radius to the same point. This makes calculating the area of the halves of the quadrilateral simple.

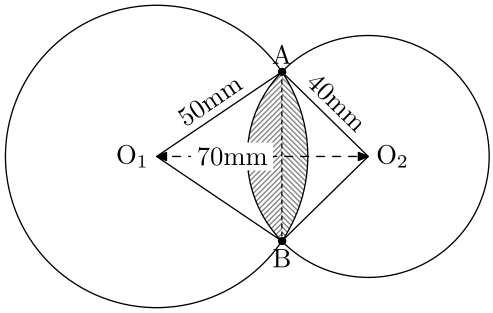
$$\begin{aligned}\text{area}_{\triangle\text{AOC}} &= \frac{1}{2}\times6\times8\\ &= 24\\ \text{area}_{\text{AOBC}} &= 48\\ \angle AOC &= \tan^{-1}\frac{8}{6}\\ &\approx 0.927\\ \angle AOB &= 2\angle AOC\\ &\approx 1.855\\ \text{area}_{\text{sector AOB}} &= \frac{1}{2}\times6^2\times1.855\\ &= 33.38\\ \text{area} &= 48-33.38\\ &= 14.62\text{cm}^2\end{aligned}$$

21. Area ABCD is equal to the area of the segment created by chord AD minus the area of the segment created by chord BC.

$$\begin{aligned}a &= \frac{1}{2}\times5^2(2-\sin2)-\frac{1}{2}\times5^2(1-\sin1)\\ &= \frac{1}{2}\times5^2\left(2-\sin2-(1-\sin1)\right)\\ &= \frac{25}{2}(1-\sin2+\sin1)\\ &= 11.65\text{cm}^2\end{aligned}$$

22. (a) $l = r\theta = 75\times0.8 = 60$cm each way, or 120cm total.

(b) BC$=2\times75\sin0.4\approx58.4$cm
Arc BC exceeds chord BC by 1.6cm.

23. 

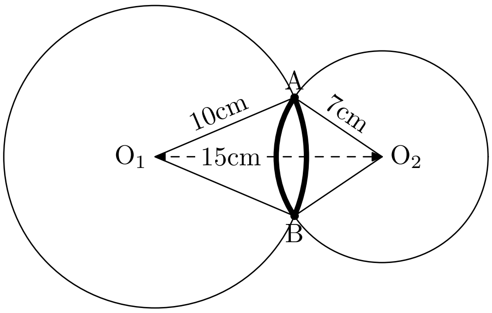
Consider $\triangle O_1AO_2$.

$$\begin{aligned}\angle AO_1O_2 &= \cos^{-1}\frac{50^2+70^2-40^2}{2\times50\times70}\\ &\approx 0.594\\ \angle AO_1B &= 2\angle AO_1O_2\\ &\approx 1.188\\ \text{segment } AO_1B &= \frac{1}{2}\times50^2(1.188-\sin1.188)\\ &\approx 325.9\end{aligned}$$

Similarly

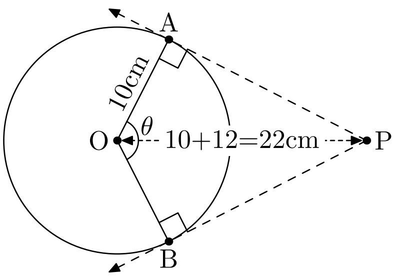
$$\begin{aligned}\angle AO_2O_1 &= \cos^{-1}\frac{40^2+70^2-50^2}{2\times40\times70}\\ &\approx 0.775\\ \angle AO_2B &= 2\angle AO_2O_1\\ &\approx 1.550\\ \text{segment } AO_2B &= \frac{1}{2}\times40^2(1.550-\sin1.550)\\ &\approx 440.5\\ \text{total area} &= 325.9+440.5\\ &\approx 770\text{mm}^2\end{aligned}$$

24.



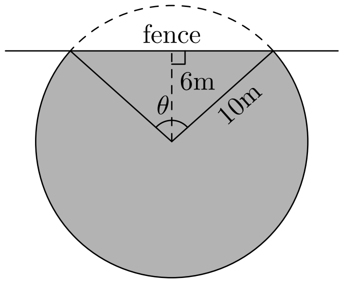
$$\begin{aligned}
\angle AO_1O_2 &= \cos^{-1} \frac{10^2+15^2-7^2}{2\times 10\times 15} \\
&\approx 0.403 \\
\angle AO_1B &= 2\angle AO_1O_2 \\
&\approx 0.805 \\
\text{arc } AO_1B &= 10 \times 0.805 \\
&\approx 8.05 \\
\angle AO_2O_1 &= \cos^{-1} \frac{7^2+15^2-10^2}{2\times 7\times 15} \\
&\approx 0.594 \\
\angle AO_2B &= 2\angle AO_2O_1 \\
&\approx 1.188 \\
\text{arc } AO_2B &= 7 \times 1.188 \\
&\approx 8.32 \\
\text{perimeter} &= 8.05 + 8.32 \\
&\approx 16.4\text{cm}
\end{aligned}$$

25.



$$\begin{aligned}
\cos\frac{\theta}{2} &= \frac{10}{10+12} \\
\theta &= 2\cos^{-1}\frac{10}{22} \\
&\approx 2.20 \\
\text{percentage} &= \frac{2.20}{2\pi} \times 100\% \\
&\approx 35\%
\end{aligned}$$

26.



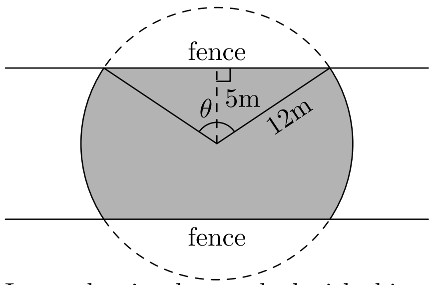
To

$$\begin{aligned}
\cos\frac{\theta}{2} &= \frac{6}{10} \\
\theta &= 2\cos^{-1}\frac{6}{10} \\
&\approx 1.85
\end{aligned}$$

find the major segment, use $2\pi - \theta$:

$$\begin{aligned}
a &= \frac{1}{2} \times 10^2 \left(2\pi - \theta - \sin(2\pi - \theta)\right) \\
&\approx 269\text{m}^2
\end{aligned}$$

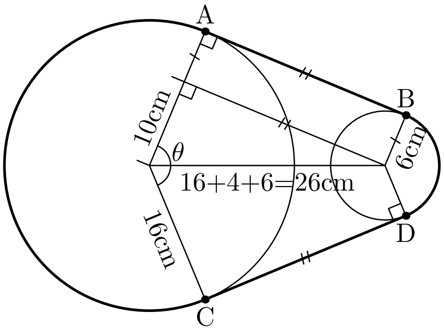
27.



It may be simplest to deal with this as the area of the circle minus the area of two identical minor segments.

$$\begin{aligned}
a_\circ &= \pi \times 12^2 \\
&\approx 452.39\text{m}^2 \\
\cos\frac{\theta}{2} &= \frac{5}{12} \\
\theta &= 2\cos^{-1}\frac{5}{12} \\
&\approx 2.28 \\
a_{\text{seg}} &= \frac{1}{2}r^2\left(\theta - \sin\theta\right) \\
&= \frac{12^2}{2}(2.28 - \sin 2.28) \\
&\approx 109.76\text{m}^2 \\
a &= a_\circ - 2a_{\text{seg}} \\
&\approx 452.39 - 2\times 109.76 \\
&\approx 233\text{m}^2
\end{aligned}$$

28.



This is similar to the Situation at the beginning of the chapter.

Straight segments AB and CD

$$\begin{aligned} AB &= \sqrt{26^2 - 10^2} \\ &= 24\text{cm} \end{aligned}$$

Angle θ

$$\begin{aligned} \cos\frac{\theta}{2} &= \frac{10}{26} \\ \theta &= 2\cos^{-1}\frac{10}{26} \\ &\approx 2.35 \end{aligned}$$

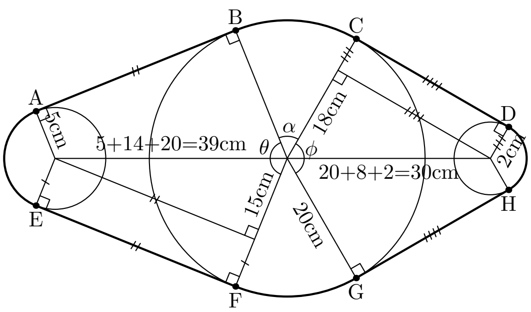
Major arc AC

$$\begin{aligned} AC &= r(2\pi - \theta) \\ &= 16(2\pi - 2.35) \\ &\approx 62.90 \end{aligned}$$

Minor arc BD

$$\begin{aligned} BD &= r\theta \\ &= 6 \times 2.35 \\ &\approx 14.11 \\ \text{Total length} &= 2 \times 24 + 62.90 + 14.11 \\ &\approx 125\text{cm} \end{aligned}$$

29.



Straight segments AB and EF

$$\begin{aligned} AB &= \sqrt{39^2 - 15^2} \\ &= 36\text{cm} \end{aligned}$$

Straight segments CD and GH

$$\begin{aligned} CD &= \sqrt{30^2 - 18^2} \\ &= 24\text{cm} \end{aligned}$$

Angle θ

$$\begin{aligned} \cos\frac{\theta}{2} &= \frac{15}{39} \\ \theta &= 2\cos^{-1}\frac{15}{39} \\ &\approx 2.35 \end{aligned}$$

Minor arc AE

$$\begin{aligned} AE &= r\theta \\ &= 5 \times 2.35 \\ &\approx 11.76 \end{aligned}$$

Angle ϕ

$$\begin{aligned} \cos\frac{\phi}{2} &= \frac{18}{30} \\ \phi &= 2\cos^{-1}\frac{18}{30} \\ &\approx 1.85 \end{aligned}$$

Minor arc DH

$$\begin{aligned} DH &= r\phi \\ &= 2 \times 1.85 \\ &\approx 3.71 \end{aligned}$$

Minor arcs BC and FG

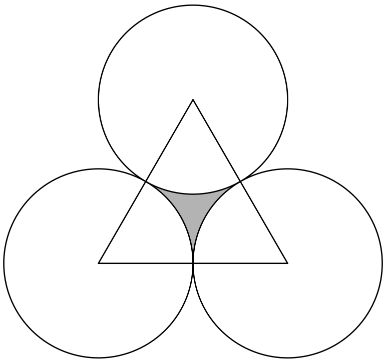
$$\begin{aligned} \alpha &= \frac{1}{2}(2\pi - \theta - \phi) \\ &\approx 1.04 \\ BG &= r\alpha \\ &= 20 \times 1.04 \\ &\approx 20.77 \\ \text{Total length} &= 2(AB + BC + CD) + AE + DH \\ &= 2(36 + 20.77 + 24) + 11.76 + 3.71 \\ &\approx 177\text{cm} \end{aligned}$$

30.

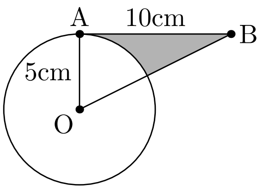
$$\begin{aligned} \text{perimeter:} \quad 2r + r\theta &= 14 \\ r\theta &= 14 - 2r \\ \theta &= \frac{14}{r} - 2 \\ \text{area:} \quad \frac{1}{2}r^2\theta &= 10 \\ \text{subst. for } \theta\text{:} \quad \frac{1}{2}r^2\left(\frac{14}{r} - 2\right) &= 10 \\ 7r - r^2 &= 10 \\ r^2 - 7r + 10 &= 0 \\ (r-5)(r-2) &= 0 \\ r &= 5 \\ \text{or } r &= 2 \\ \text{for } r = 5,\ \theta &= \frac{14}{5} - 2 \\ &= 0.8 \text{ (an acute angle)} \\ \text{for } r = 2,\ \theta &= \frac{14}{2} - 2 \\ &= 5 \text{ (a reflex angle)} \end{aligned}$$

(a) Radius = 5 cm.

(b) Radius = 2 cm.

31. 

$$\begin{aligned}
a_{\triangle} &= \frac{1}{2} \times 10 \times 10 \sin \frac{\pi}{3} \\
&= \frac{50\sqrt{3}}{2} \\
&= 25\sqrt{3} \\
a_{\text{sector}} &= \frac{1}{2} \times 5^2 \times \frac{\pi}{3} \\
&= \frac{25\pi}{6} \\
a_{\text{shaded}} &= a_{\triangle} - 3a_{\text{sector}} \\
&= 25\sqrt{3} - 3 \times \frac{25\pi}{6} \\
&= 25\sqrt{3} - \frac{25\pi}{2} \\
&= 25\left(\sqrt{3} - \frac{\pi}{2}\right) \text{cm}^2
\end{aligned}$$

32. 

$$\begin{aligned}
a_{\triangle} &= \frac{1}{2} \times 10 \times 5 \\
&= 25 \\
\theta &= \tan^{-1} \frac{10}{5} \\
&\approx 1.11 \\
a_{\text{sector}} &= \frac{1}{2} \times 5^2 \times 1.11 \\
&\approx 13.84 \\
a_{\text{shaded}} &= a_{\triangle} - a_{\text{sector}} \\
&= 25 - 13.84 \\
&\approx 11.2\text{cm}^2
\end{aligned}$$

33. r is the slant height of the cone, and θ is such that the arc length of the sector equals the circumference of the base of the cone.

$$\begin{aligned}
r &= \sqrt{28^2 + 8^2} \\
&= 4\sqrt{53} \\
&\approx 29.1\text{cm} \\
r\theta &= 2\pi \times 8 \\
\theta &= \frac{16\pi}{4\sqrt{53}} \\
&\approx 1.73^{\text{RADIANS}}
\end{aligned}$$

34. First, find the area of the major segment, then find the capacity:

$$\begin{aligned}
\cos \frac{\theta}{2} &= \frac{10}{60} \\
\theta &= 2\cos^{-1} \frac{1}{6} \\
&\approx 2.81 \\
a &= \frac{1}{2} \times 60^2 \left(2\pi - \theta - \sin(2\pi - \theta)\right) \\
&\approx 6849\text{cm}^2 \\
V &= al \\
&= 6849 \times 120 \\
&\approx 821915\text{cm}^3 \\
&\approx 822\text{L}
\end{aligned}$$

35. (a)

$$\begin{aligned}
a_{\text{I}} &= \frac{1}{2}bh \\
&= \frac{1}{2} \times 15 \times 40 \\
&= 300\text{cm}^2 \\
a_{\text{II}} &= a_{\text{I}} = 300\text{cm}^2 \\
a_{\text{III}} &= a_{\text{I}} + a_{\text{II}} = 600\text{cm}^2
\end{aligned}$$

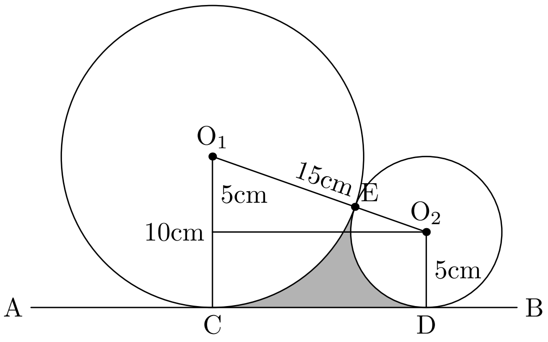
for segment IV:

$$\begin{aligned}
r &= \sqrt{40^2 + 15^2} \\
&= 5\sqrt{73} \\
&\approx 42.72 \\
\theta &= 2\tan^{-1} \frac{15}{40} \\
&\approx 0.718 \\
a_{\text{IV}} &= \frac{1}{2}r^2\theta - a_{III} \\
&= \frac{1}{2} \times 1825 \times 0.718 - 600 \\
&\approx 55\text{cm}^2
\end{aligned}$$

(b)

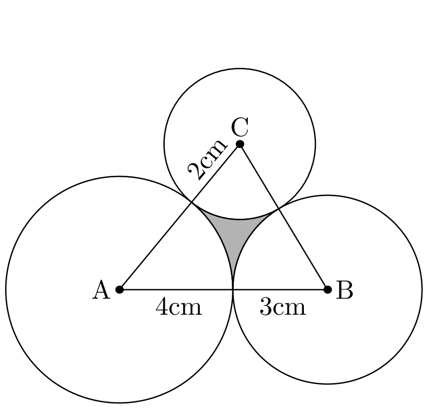
$$\begin{aligned}
l = 40 \times 2 &\quad +30 \times 2 \\
+ 5\sqrt{73} \times 2 &\quad +5\sqrt{73} \times 0.718 \\
\approx 256\text{cm}
\end{aligned}$$

36.



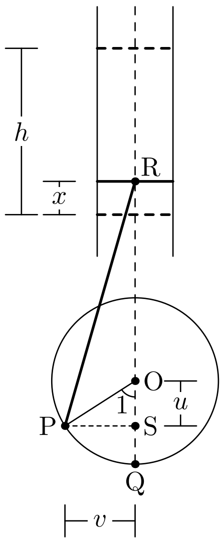
$$
\begin{aligned}
CD &= \sqrt{15^2 - 5^2} \\
&= 10\sqrt{2} \\
a_{\mathrm{O_1O_2DC}} &= \frac{1}{2}\mathrm{CD}(r_1 + r_2) \\
&= \frac{1}{2} \times 10\sqrt{2}(10+5) \\
&= 75\sqrt{2} \\
&\approx 106.07 \\
\cos\angle\mathrm{CO_1E} &= \frac{5}{15} \\
\angle\mathrm{CO_1E} &\approx 1.23 \\
a_{\mathrm{CO_1E}} &= \frac{1}{2} \times 10^2 \times 1.23 \\
&\approx 61.55 \\
\sin\angle\mathrm{DO_2E} - \frac{\pi}{4} &= \frac{5}{15} \\
\angle\mathrm{DO_2E} &\approx 0.34 + \frac{\pi}{4} \\
&\approx 1.91 \\
a_{\mathrm{DO_2E}} &= \frac{1}{2} \times 5^2 \times 1.91 \\
&\approx 23.88 \\
a_{\mathrm{shaded}} &= a_{\mathrm{O_1O_2DC}} - a_{\mathrm{CO_1E}} - a_{\mathrm{dO_2E}} \\
&\approx 106.07 - 61.55 - 23.88 \\
&\approx 20.64\mathrm{cm}^2
\end{aligned}
$$

37.



$$
\begin{aligned}
\angle\mathrm{A} &= \cos^{-1}\frac{6^2 + 7^2 - 5^2}{2 \times 6 \times 7} \\
&\approx 0.775 \\
\angle\mathrm{B} &= \cos^{-1}\frac{5^2 + 7^2 - 6^2}{2 \times 5 \times 7} \\
&\approx 0.997 \\
\angle\mathrm{C} &= \cos^{-1}\frac{6^2 + 5^2 - 7^2}{2 \times 6 \times 5} \\
&\approx 1.369 \\
a_{\triangle} &= \frac{1}{2}ab\sin\mathrm{C} \\
&= \frac{1}{2} \times 5 \times 6\sin 1.369 \\
&\approx 14.70 \\
a_{\text{sector A}} &= \frac{1}{2} \times 4^2 \times 0.775 \\
&\approx 6.20 \\
a_{\text{sector B}} &= \frac{1}{2} \times 3^2 \times 0.997 \\
&\approx 4.49 \\
a_{\text{sector C}} &= \frac{1}{2} \times 2^2 \times 1.369 \\
&\approx 2.74 \\
a_{\text{shaded}} &= a_{\triangle} - a_{\text{sector A}} - a_{\text{sector B}} a_{\text{sector C}} \\
&= 14.70 - 6.20 - 4.49 - 2.74 \\
&\approx 1.27 \\
\frac{a_{\text{shaded}}}{a_{\triangle}} &= \frac{1.27}{14.70} \\
&= 8.6\%
\end{aligned}
$$

38.



The lowest position of the piston is r above the top of the wheel. Therefore it is $3r$ above the bottom of the wheel. Thus the fixed length of the drive rod PR= $3r$.

It should be clear that $h = 2r$ since at the low position the arm goes straight down from the piston to the bottom of the wheel, and at the high position the arm goes straight down from the piston to the top of the wheel.

If arc length PQ is equal to r, the angle it subtends is $\angle\mathrm{POQ}=1^{\mathrm{RADIAN}}$.

The height of point P relative to point O is $u = r\cos 1$.

Similarly the horizontal position of point P is $v = -r\sin 1$.

The length from point S to R can be determined by Pythagoras' Theorem:

$$\begin{aligned}\text{SR} &= \sqrt{(3r)^2 - (r\sin 1)^2}\\ &= \sqrt{9r^2 - r^2\sin^2 1}\\ &= r\sqrt{9-\sin^2 1}\\ &= 2.88r\end{aligned}$$

$$\begin{aligned}x &= \text{SR} - 2r - u\\ &= 2.88r - 2r - r\cos 1\\ &= r(2.88 - 2 - \cos 1)\\ &= 0.33r\\ \frac{x}{h} &= \frac{0.33r}{2r}\\ &\approx 17\%\end{aligned}$$

Miscellaneous Exercise 5

1. No worked solution necessary.
2. No worked solution necessary.
3. (a) $x < 2.5$: $2x - 5 < 0$; $4 - x > 0$

$$\begin{aligned}-(2x-5) &= 4 - x\\ -2x + 5 &= 4 - x\\ x &= 1\end{aligned}$$

(which is in the part of the domain we are considering, so we do not reject it.)

$2.5 \le x < 4$: $2x - 5 \ge 0$; $4 - x > 0$

$$\begin{aligned}2x - 5 &= 4 - x\\ 3x &= 9\\ x &= 3\end{aligned}$$

$x \ge 4$: $2x - 5 \ge 0$; $4 - x \le 0$

$$\begin{aligned}2x - 5 &= -(4 - x)\\ x &= 1\end{aligned}$$

We don't really need to do the third part (which yields a solution outside the part of the domain we're considering), as there can only be two solutions. (You can see that if you graph the left and right hand sides of the equation and see where they intersect.)

Solution: $x = 1$ or $x = 3$.

(b) $x < 2.5$: $2x - 5 < 0$; $4 - x > 0$

$$\begin{aligned}-(2x-5) &< 4 - x\\ -2x + 5 &< 4 - x\\ x &> 1\end{aligned}$$

(which is in the part of the domain we are considering, so we have $1 < x < 2.5$.)

$2.5 \le x < 4$: $2x - 5 \ge 0$; $4 - x > 0$

$$\begin{aligned}2x - 5 &< 4 - x\\ 3x &< 9\\ x &< 3\end{aligned}$$

(which is in the part of the domain we are considering, so we have $2.5 \le x < 4$.)

Combining the two parts of solution we get:

Solution: $1 < x < 3$.

(c) This inequality must be true where the previous inequality is false, i.e.:

Solution: $x \le 1$ or $x \ge 3$.

There is another way we could look at this. We know the left and right hand sides are equal at 1 and 3, so the inequality must be true either between 1 and 3 or less than 1/greater than 3. We can decide which by testing a suitable value: substitute (for example) 2 for x and decide whether the inequality holds true. Here $|2 \times 2 - 5| = 1$ and $|4 - 2| = 2$ so LHS<RHS and the second inequality holds for the interval $1 < x < 3$.

4. $l = r\theta = \frac{40}{2} \times 3 = 60\text{cm/s}$
5. (a) $$\begin{aligned}\text{BC} &= \sqrt{\text{AC}^2 - \text{AB}^2}\\ &= \sqrt{10^2 - 6^2}\\ &= 8\text{cm}\end{aligned}$$

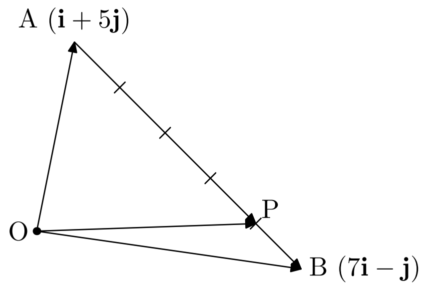
(b) $$\begin{aligned}\text{DB} &= \sqrt{\text{DA}^2 + \text{AB}^2}\\ &= \sqrt{15^2 + 6^2}\\ &= \sqrt{261}\\ &= 3\sqrt{29}\text{cm}\end{aligned}$$

(c) $$\begin{aligned}\cos\angle\text{CAB} &= \frac{AB}{AC}\\ \angle\text{CAB} &= \cos^{-1}\frac{AB}{AC}\\ &= \cos^{-1}\frac{6}{10}\\ &\approx 53^\circ\end{aligned}$$

(d) We want ∠FAE.

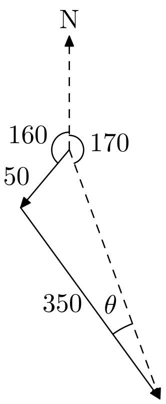
$$\begin{aligned}\tan\angle\text{FAE} &= \frac{FE}{EA}\\ \angle\text{FAE} &= \tan^{-1}\frac{FE}{EA}\\ &= \tan^{-1}\frac{8}{3\sqrt{29}}\\ &\approx 26^\circ\end{aligned}$$

6.



$$\begin{aligned}\overrightarrow{\text{AP}} &= \frac{4}{5}\overrightarrow{\text{AB}}\\ \overrightarrow{\text{OP}}-\overrightarrow{\text{OA}} &= \frac{4}{5}(\overrightarrow{\text{OB}}-\overrightarrow{\text{OA}})\\ \overrightarrow{\text{OP}} &= \frac{4}{5}(\overrightarrow{\text{OB}}-\overrightarrow{\text{OA}})+\overrightarrow{\text{OA}}\\ &= \frac{4}{5}\overrightarrow{\text{OB}}+\frac{1}{5}\overrightarrow{\text{OA}}\\ &= 0.8(7\mathbf{i}-\mathbf{j})+0.2(\mathbf{i}+5\mathbf{j})\\ &= (5.6\mathbf{i}-0.8\mathbf{j})+(0.2\mathbf{i}+\mathbf{j})\\ &= 5.8\mathbf{i}+0.2\mathbf{j}\end{aligned}$$

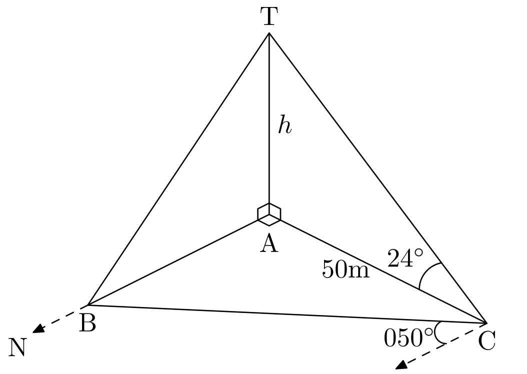
7.



$$\begin{aligned}\frac{\sin\theta}{50} &= \frac{\sin 30^\circ}{350}\\ \theta &= \sin^{-1}\frac{50\sin 30}{350}\\ &\approx 4.1^\circ\end{aligned}$$

The plane must fly on a bearing of $170-4=166^\circ$.

8.



(a)
$$\begin{aligned}\tan 24^\circ &= \frac{h}{50}\\ h &= 50\tan 24^\circ\\ &\approx 22\text{m}\end{aligned}$$

(b)
$$\begin{aligned}\angle\text{ACB} &= 90^\circ-50^\circ\\ &= 40^\circ\\ \cos 40^\circ &= \frac{50}{\text{BC}}\\ \text{BC} &= \frac{50}{\cos 40^\circ}\\ &\approx 65\text{m}\end{aligned}$$

(c)
$$\begin{aligned}\tan 40^\circ &= \frac{\text{AB}}{50}\\ \text{AB} &= 50\tan 40^\circ\\ &\approx 42\text{m}\end{aligned}$$

(d)
$$\begin{aligned}\tan\angle\text{TBA} &= \frac{h}{AB}\\ \angle\text{TBA} &= \tan^{-1}\frac{22.26}{41.95}\\ &\approx 28^\circ\end{aligned}$$

9. $\angle\text{BOA} = 45^\circ$

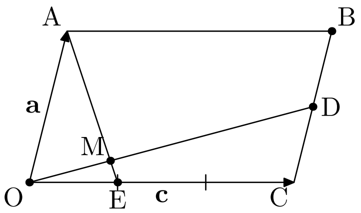
$$\begin{aligned}\text{BA} &= \sqrt{10^2+10^2-2\times 10\times 10\sin 45^\circ}\\ &= 10\left(\sqrt{2-\sqrt{2}}\right)\\ &\approx 7.65\text{m}\end{aligned}$$

By the similarity of the triangles BOA and JOI:

$$\begin{aligned}\text{JI} &= \frac{3}{5}\text{BA}\\ &\approx 4.59\text{m}\end{aligned}$$

10.
$$\begin{aligned}
\mathbf{a} &= k\mathbf{c}\\
k &= \frac{|\mathbf{b}|}{|\mathbf{c}|}\\
&= \frac{\sqrt{7^2+24^2}}{\sqrt{3^2+4^2}}\\
&= \frac{25}{5}\\
&= 5\\
\mathbf{a} &= 5(3\mathbf{i}-4\mathbf{j})\\
&= 15\mathbf{i}-20\mathbf{j}\\
\mathbf{a}+\mathbf{b} &= 15\mathbf{i}-20\mathbf{j}-7\mathbf{i}+24\mathbf{j}\\
&= 8\mathbf{i}+4\mathbf{j}\\
|\mathbf{a}+\mathbf{b}| &= \sqrt{8^2+4^2}\\
&= \sqrt{80}\\
&= 4\sqrt{5}
\end{aligned}$$

11.



$$\begin{aligned}
\overrightarrow{AE} &= -\mathbf{a}+\frac{1}{3}\mathbf{c}\\
\overrightarrow{AM} &= h\overrightarrow{AE}\\
&= -h\mathbf{a}+\frac{h}{3}\mathbf{c}\\
\overrightarrow{OM} &= \mathbf{a}+\overrightarrow{AM}\\
&= \mathbf{a}-h\mathbf{a}+\frac{h}{3}\mathbf{c}\\
&= (1-h)\mathbf{a}+\frac{h}{3}\mathbf{c}\\
\overrightarrow{OD} &= \mathbf{c}+\frac{1}{2}\mathbf{a}\\
\overrightarrow{OM} &= k\overrightarrow{OD}\\
&= k(\mathbf{c}+\frac{1}{2}\mathbf{a})\\
&= \frac{k}{2}\mathbf{a}+k\mathbf{c}
\end{aligned}$$

$$\therefore \quad (1-h)\mathbf{a}+\frac{h}{3}\mathbf{c} = \frac{k}{2}\mathbf{a}+k\mathbf{c}$$

$$(1-h)\mathbf{a}-\frac{k}{2}\mathbf{a} = k\mathbf{c}-\frac{h}{3}\mathbf{c}$$

$$\left(1-h-\frac{k}{2}\right)\mathbf{a} = \left(k-\frac{h}{3}\right)\mathbf{c}$$

$$\begin{aligned}
\therefore \quad 1-h-\frac{k}{2} &= 0\\
h+\frac{k}{2} &= 1\\
2h+k &= 2 && ①\\
\text{and } k-\frac{h}{3} &= 0\\
-h+3k &= 0 && ②\\
7k &= 2 && (①+2\times②)\\
k &= \frac{2}{7}\\
-h+3\left(\frac{2}{7}\right) &= 0 && (\text{subst into } ②)\\
h &= \frac{6}{7}
\end{aligned}$$

12. (a)

$$\begin{aligned}
\overrightarrow{OA} &= (19\mathbf{i}-6\mathbf{j})+t(-2\mathbf{i}+5\mathbf{j})\\
&= (19-2t)\mathbf{i}+(-6+5t)\mathbf{j}
\end{aligned}$$

$$\begin{aligned}
|\overrightarrow{OA}| &= 25\\
(19-2t)^2+(-6+5t)^2 &= 25^2\\
361-76t+4t^2+36-60t+25t^2 &= 625\\
29t^2-136t+397 &= 625\\
29t^2-136t-228 &= 0\\
(t-6)(29t+38) &= 0\\
t &= 6\text{s}
\end{aligned}$$

(b)
$$\begin{aligned}
\overrightarrow{AB} &= \overrightarrow{OB}-\overrightarrow{OA}\\
&= (5\mathbf{i}+\mathbf{j})-((19-2t)\mathbf{i}+(-6+5t)\mathbf{j})\\
&= (5-19+2t)\mathbf{i}+(1+6-5t)\mathbf{j}\\
&= (-14+2t)\mathbf{i}+(7-5t)\mathbf{j}
\end{aligned}$$

$$|\overrightarrow{OA}| = |\overrightarrow{AB}|$$

$$\begin{aligned}
\therefore \quad 29t^2-136t+397 &= (-14+2t)^2+(7-5t)^2\\
29t^2-136t+397 &= 196-56t+4t^2\\
&\quad +49-70t+25t\\
29t^2-136t+397 &= 29t^2-126t+245\\
10t &= 152\\
t &= 15.2\text{s}
\end{aligned}$$