Chapter 1

Exercise 1A

1. The initial case, where $n = 1$,

$$1 = \frac{1}{2}(1)(1+1)$$

is true.

Assume the statement is true for $n = k$, i.e.

$$1 + 2 + 3 + 4 + \ldots + k = \frac{1}{2}k(k+1)$$

Then for $n = k + 1$

$$\begin{aligned} 1 + 2 + 3 + 4 + &\ldots + k + (k+1) \\ &= \frac{1}{2}k(k+1) + (k+1) \\ &= (\frac{1}{2}k + 1)(k+1) \\ &= \frac{1}{2}(k+2)(k+1) \\ &= \frac{1}{2}(k+1)\left((k+1)+1\right) \end{aligned}$$

Thus if the statement is true for $n = k$ it is also true for $n = k + 1$.

Since the statement is true for $n = 1$ it follows by induction that it is true for all integer $n \geq 1$. □

2. The initial case, where $n = 1$:

$$\begin{aligned} \text{L.H.S.} &= 1(1+1) \\ &= 2 \\ \text{R.H.S.} &= \frac{1}{3}(1+1)(1+2) \\ &= 2 \\ &= \text{L.H.S.} \end{aligned}$$

The statement is true for the initial case.

Assume the statement is true for $n = k$, i.e.

$$1\times2+2\times3+3\times4+\ldots+k(k+1) = \frac{k}{3}(k+1)(k+2)$$

Then for $n = k + 1$:

$$\begin{aligned} 1\times2+2\times3+3\times4+&\ldots+k(k+1)+(k+1)(k+2) \\ &= \frac{k}{3}(k+1)(k+2) + (k+1)(k+2) \\ &= (\frac{k}{3}+1)(k+1)(k+2) \\ &= \frac{1}{3}(k+3)(k+1)(k+2) \\ &= \frac{k+1}{3}(k+2)(k+3) \\ &= \frac{k+1}{3}\left((k+1)+1\right)\left((k+1)+2\right) \end{aligned}$$

Thus if the statement is true for $n = k$ it is also true for $n = k + 1$.

Hence since the statement is true for $n = 1$ it follows by induction that it is true for all integer $n \geq 1$. □

3. The initial case, where $n = 1$ is given:

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^1) = 1$$

The statement is true for the initial case.

Assume the statement is true for $n = k$, i.e.

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^k) = kx^{k-1}$$

Then for $n = k + 1$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x}(x^{k+1}) &= \frac{\mathrm{d}}{\mathrm{d}x}(xx^k) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}(x)(x^k) + (x)\left(\frac{\mathrm{d}}{\mathrm{d}x}(x^k)\right) \\ &= x^k + x\left(kx^{k-1}\right) \\ &= x^k + kx^k \\ &= (k+1)x^k \\ &= (k+1)x^{(k+1)-1} \end{aligned}$$

Thus if the statement is true for $n = k$ it is also true for $n = k + 1$.

Hence since the statement is true for $n = 1$ it follows by induction that it is true for all integer $n \geq 1$. □

4. The initial case, where $n = 1$:

$$2 = 2^2 - 2$$

The statement is true for the initial case.

Assume the statement is true for $n = k$, i.e.

$$2 + 4 + 8 + \ldots + 2^k = 2^{k+1} - 2$$

Then for $n = k + 1$

$$\begin{aligned} 2 + 4 + 8 + \ldots + 2^k + 2^{k+1} &= 2^{k+1} - 2 + 2^{k+1} \\ &= 2(2^{k+1}) - 2 \\ &= 2^{(k+1)+1} - 2 \end{aligned}$$

Thus if the statement is true for $n = k$ it is also true for $n = k + 1$.

Hence since the statement is true for $n = 1$ it follows by induction that it is true for all integer $n \geq 1$. □

5. The initial case, where $n = 1$:

$$\begin{aligned} \text{L.H.S.} &= 1(1+1)^3 \\ &= 1 \\ \text{R.H.S.} &= \frac{1^2}{4}(1+1)(1+2)^2 \\ &= 1 \\ &= \text{L.H.S.} \end{aligned}$$

The statement is true for the initial case.

Assume the statement is true for $n = k$, i.e.

$$1^3 + 2^3 + 3^3 + 4^3 + \ldots + k^3 = \frac{k^2}{4}(k+1)^2$$

Then for $n = k + 1$

$$\begin{aligned}1^3 + 2^3 + 3^3 + 4^3 + \ldots + k^3 + (k+1)^3 \\ &= \frac{k^2}{4}(k+1)^2 + (k+1)^3 \\ &= \frac{k^2}{4}(k+1)^2 + (k+1)(k+1)^2 \\ &= \frac{k^2 + 4(k+1)}{4}(k+1)^2 \\ &= \frac{k^2 + 4k + 4}{4}(k+1)^2 \\ &= \frac{(k+2)^2}{4}(k+1)^2 \\ &= \frac{(k+1)^2}{4}(k+2)^2 \\ &= \frac{(k+1)^2}{4}((k+1)+1)^2\end{aligned}$$

Thus if the statement is true for $n = k$ it is also true for $n = k + 1$.

Hence since the statement is true for $n = 1$ it follows by induction that it is true for all integer $n \geq 1$. □

6. (a) For $n = 2$, $(2n - 1) = 4 - 1 = 3$ and $n^2 = 4$ hence

$$1 + 3 = 4$$

is consistent with the rule.
For $n = 3$, $(2n - 1) = 6 - 1 = 5$ and $n^2 = 9$ hence

$$1 + 3 + 5 = 9$$

is consistent with the rule.
Verify the other statements similarly.

(b) The initial case, where $n = 1$: $2n - 1 = 1$ and

$$1 = 1^2$$

The statement is true for the initial case.
Assume the statement is true for $n = k$, i.e.

$$1 + 3 + 5 + \ldots + (2k - 1) = k^2$$

Then for $n = k + 1$

$$\begin{aligned}1 + 3 + 5 + \ldots + (2k-1) + (2(k+1) - 1) \\ &= k^2 + (2(k+1) - 1) \\ &= k^2 + 2k + 2 - 1 \\ &= k^2 + 2k + 1 \\ &= (k+1)^2\end{aligned}$$

Thus if the statement is true for $n = k$ it is also true for $n = k + 1$.
Hence since the statement is true for $n = 1$ it follows by induction that it is true for all integer $n \geq 1$. □

7. The initial case, where $n = 1$:

$$\frac{1}{2} = \frac{2-1}{2}$$

The statement is true for the initial case.

Assume the statement is true for $n = k$, i.e.

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{1}{2^k} = \frac{2^k - 1}{2^k}$$

Then for $n = k + 1$

$$\begin{aligned}\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{1}{2^k} + \frac{1}{2^{k+1}} \\ &= \frac{2^k - 1}{2^k} + \frac{1}{2^{k+1}} \\ &= \frac{2(2^k - 1)}{2^{k+1}} + \frac{1}{2^{k+1}} \\ &= \frac{2(2^k - 1) + 1}{2^{k+1}} \\ &= \frac{2^{k+1} - 2 + 1}{2^{k+1}} \\ &= \frac{2^{k+1} - 1}{2^{k+1}}\end{aligned}$$

Thus if the statement is true for $n = k$ it is also true for $n = k + 1$.

Hence since the statement is true for $n = 1$ it follows by induction that it is true for all integer $n \geq 1$. □

8. The initial case, where $n = 1$:

$$\frac{1}{1(1+1)} = \frac{1}{1+1}$$

The statement is true for the initial case.

Assume the statement is true for $n = k$, i.e.

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \ldots + \frac{1}{k(k+1)} = \frac{k}{k+1}$$

Then for $n = k + 1$

$$\begin{aligned}\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \ldots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} \\ &= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} \\ &= \frac{k(k+2)}{(k+1)(k+2)} + \frac{1}{(k+1)(k+2)} \\ &= \frac{k(k+2) + 1}{(k+1)(k+2)} \\ &= \frac{k^2 + 2k + 1}{(k+1)(k+2)} \\ &= \frac{(k+1)^2}{(k+1)(k+2)} \\ &= \frac{k+1}{k+2} \\ &= \frac{k+1}{(k+1)+1}\end{aligned}$$

Thus if the statement is true for $n = k$ it is also true for $n = k + 1$.

Hence since the statement is true for $n = 1$ it follows by induction that it is true for all integer $n \geq 1$. $\square$

9. The initial case, where $n = 1$:

$$\begin{aligned} \text{L.H.S.} &= 1(1+2)(1+4) \\ &= 10 \\ \text{R.H.S.} &= \frac{1}{4}(1+1)(1+4)(1+5) \\ &= 10 \\ &= \text{L.H.S.} \end{aligned}$$

The statement is true for the initial case.

Assume the statement is true for $n = k$, i.e.

$$\begin{aligned} 1 \times 3 \times 5 + 2 \times 4 \times 6 + \ldots + k(k+2)(k+4) \\ = \frac{k}{4}(k+1)(k+4)(k+5) \end{aligned}$$

Then for $n = k + 1$

$$\begin{aligned} &1 \times 3 \times 5 + 2 \times 4 \times 6 + \ldots \\ &\quad + k(k+2)(k+4) + (k+1)(k+3)(k+5) \\ &= \frac{k}{4}(k+1)(k+4)(k+5) + (k+1)(k+3)(k+5) \\ &= (k+1)(k+5)\left(\frac{k}{4}(k+4) + (k+3)\right) \\ &= \frac{k+1}{4}(k+5)\left(k(k+4) + 4(k+3)\right) \\ &= \frac{k+1}{4}\left((k+1)+4\right)\left(k^2 + 4k + 4k + 12\right) \\ &= \frac{k+1}{4}\left((k+1)+4\right)\left(k^2 + 8k + 12\right) \\ &= \frac{k+1}{4}\left((k+1)+4\right)(k+2)(k+6) \\ &= \frac{k+1}{4}\left((k+1)+4\right)\left((k+1)+1\right)\left((k+1)+5\right) \\ &= \frac{k+1}{4}\left((k+1)+1\right)\left((k+1)+4\right)\left((k+1)+5\right) \end{aligned}$$

Thus if the statement is true for $n = k$ it is also true for $n = k + 1$.

Hence since the statement is true for $n = 1$ it follows by induction that it is true for all integer $n \geq 1$. $\square$

10. The initial case, where $n = 1$: $(x - 1)$ is a factor of $x^1 - 1$ since $x - 1 = x^1 - 1$.

The statement is true for the initial case.

Assume the statement is true for $n = k$, i.e.

$$x^k - 1 = a(x - 1)$$

Then for $n = k + 1$

$$\begin{aligned} x^{k+1} - 1 &= x(x^k) - 1 \\ &= x(x^k - 1 + 1) - 1 \\ &= x(x^k - 1) + x - 1 \\ &= ax(x-1) + (x-1) \\ &= (ax+1)(x-1) \end{aligned}$$

Thus if the statement is true for $n = k$ it is also true for $n = k + 1$.

Hence since the statement is true for $n = 1$ it follows by induction that it is true for all integer $n \geq 1$. $\square$

11. The initial case here is where $n = 7$, the first integer value satisfying $n > 6$:

$$\begin{aligned} \text{L.H.S.} &= 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \\ &= 5040 \\ \text{R.H.S.} &= 3^7 \\ &= 2187 \\ 5040 &> 2187 \end{aligned}$$

The statement is true for the initial case.

Assume the statement is true for $n = k;\ k > 6$, i.e.

$$1 \times 2 \times 3 \times 4 \times \ldots \times k \geq 3^k$$

Then for $n = k + 1$

$$\begin{aligned} 1 \times 2 \times 3 \times 4 \times \ldots \times k(k+1) &\geq 3^k(k+1) \\ 3^k(k+1) &= 3^{k+1}\frac{k+1}{3} \\ \text{Now} \qquad k &> 6 \\ k+1 &> 7 \\ \frac{k+1}{3} &> 1 \\ \therefore \qquad 3^k(k+1) &> 3^{k+1} \\ \therefore \qquad 1 \times 2 \times 3 \times 4 \times \ldots \times k(k+1) &> 3^{k+1} \end{aligned}$$

Thus if the statement is true for $n = k$ it is also true for $n = k + 1$.

Hence since the statement is true for $n = 7$ it follows by induction that it is true for all integer $n > 6$. $\square$

12. The initial case, where $n = 1$:

$$\begin{aligned} 7^1 + 2 \times 13^1 &= 7 + 26 \\ &= 33 \\ &= 3 \times 11 \end{aligned}$$

The statement is true for the initial case.

Assume the statement is true for $n = k$, i.e.

$$7^k + 2 \times 13^k = 3a,\ a \in \mathbb{I}$$

Then for $n = k + 1$

$$\begin{aligned} 7^{k+1} + 2 \times 13^{k+1} &= 7 \times 7^k + 13 \times 2 \times 13^k \\ &= 7 \times 7^k + (7+6) \times 2 \times 13^k \\ &= 7 \times 7^k + 7 \times 2 \times 13^k + 12 \times 13^k \\ &= 7(7^k + 2 \times 13^k) + 3(4 \times 13^k) \\ &= 7(3a) + 3(4 \times 13^k) \\ &= 3(7a + 4 \times 13^k) \end{aligned}$$

Thus if the statement is true for $n = k$ it is also true for $n = k + 1$.

Hence since the statement is true for $n = 1$ it follows by induction that it is true for all integer $n \geq 1$. $\square$

13. The initial case, where $n = 1$:

$$\begin{aligned}\text{L.H.S.} &= 2\\ \text{R.H.S.} &= \frac{2}{3}(1+(-1)^{1+1}2^1\\ &= \frac{2}{3}(1+2)\\ &= 2\\ &= \text{L.H.S.}\end{aligned}$$

The statement is true for the initial case.

Assume the statement is true for $n = k$, i.e.

$$2-4+8-16+\ldots+(-1)^{k+1}2^k = \frac{2}{3}(1+(-1)^{k+1}2^k)$$

Then for $n = k+1$

$$\begin{aligned}&2-4+8-16+\ldots+(-1)^{k+1}2^k+(-1)^{k+2}2^{k+1}\\ &= \frac{2}{3}(1+(-1)^{k+1}2^k)+(-1)^{k+2}2^{k+1}\\ &= \frac{2}{3}(1+(-1)^{k+1}2^k)+(-1)(-1)^{k+1}(2)2^k\\ &= \frac{2}{3}(1+(-1)^{k+1}2^k)-2(-1)^{k+1}2^k\\ &= 2\left(\frac{1+(-1)^{k+1}2^k}{3}-(-1)^{k+1}2^k\right)\\ &= 2\left(\frac{1+(-1)^{k+1}2^k}{3}-\frac{3(-1)^{k+1}2^k}{3}\right)\\ &= 2\left(\frac{1+(-1)^{k+1}2^k-3(-1)^{k+1}2^k}{3}\right)\\ &= 2\left(\frac{1-2(-1)^{k+1}2^k}{3}\right)\\ &= 2\left(\frac{1-(-1)^{k+1}2^{k+1}}{3}\right)\\ &= 2\left(\frac{1+(-1)(-1)^{k+1}2^{k+1}}{3}\right)\\ &= 2\left(\frac{1+(-1)^{(k+1)+1}2^{k+1}}{3}\right)\\ &= \frac{2}{3}(1+(-1)^{(k+1)+1}2^{k+1})\end{aligned}$$

Thus if the statement is true for $n = k$ it is also true for $n = k+1$.

Hence since the statement is true for $n = 1$ it follows by induction that it is true for all integer $n \geq 1$. □

Miscellaneous Exercise 1

1. (a) $$\begin{aligned}(7+3\mathrm{i})(7-3\mathrm{i}) &= 7^2-(3\mathrm{i})^2\\ &= 49+9\\ &= 58\end{aligned}$$

   (b) $$\begin{aligned}(5+\mathrm{i})(5-1\mathrm{i}) &= 5^2-(\mathrm{i})^2\\ &= 25+1\\ &= 26\end{aligned}$$

   (c) $$\begin{aligned}(3+2\mathrm{i})(2-3\mathrm{i}) &= 6-9\mathrm{i}+4\mathrm{i}-6\mathrm{i}^2\\ &= 6-5\mathrm{i}+6\\ &= 12-5\mathrm{i}\end{aligned}$$

   (d) $$\begin{aligned}(1-5\mathrm{i})^2 &= 1-10\mathrm{i}+25\mathrm{i}^2\\ &= 1-10\mathrm{i}-25\\ &= -24-10\mathrm{i}\end{aligned}$$

   (e) $$\begin{aligned}\frac{3-2\mathrm{i}}{2+\mathrm{i}} &= \frac{(3-2\mathrm{i})(2-\mathrm{i})}{(2+\mathrm{i})(2-\mathrm{i})}\\ &= \frac{6-3\mathrm{i}-4\mathrm{i}+2\mathrm{i}^2}{4-\mathrm{i}^2}\\ &= \frac{6-7\mathrm{i}-2}{4+1}\\ &= \frac{4-7\mathrm{i}}{5}\\ &= 0.8-1.4\mathrm{i}\end{aligned}$$

   (f) $$\begin{aligned}\frac{1+2\mathrm{i}}{3-4\mathrm{i}} &= \frac{(1+2\mathrm{i})(3+4\mathrm{i})}{(3-4\mathrm{i})(3+4\mathrm{i})}\\ &= \frac{3+4\mathrm{i}+6\mathrm{i}+8\mathrm{i}^2}{9-16\mathrm{i}^2}\\ &= \frac{3+10\mathrm{i}-8}{9+16}\\ &= \frac{-5+10\mathrm{i}}{25}\\ &= \frac{-1+2\mathrm{i}}{5}\\ &= -0.2+0.4\mathrm{i}\end{aligned}$$

2. (a) $$\begin{aligned}z+w &= 3-4\mathrm{i}-4+5\mathrm{i}\\ &= -1+\mathrm{i}\end{aligned}$$

   (b) $$\begin{aligned}zw &= (3-4\mathrm{i})(-4+5\mathrm{i})\\ &= -12+15\mathrm{i}+16\mathrm{i}-20\mathrm{i}^2\\ &= -12+31\mathrm{i}+20\\ &= 8+31\mathrm{i}\end{aligned}$$

   (c) $\bar{z} = 3+4\mathrm{i}$

   (d) $$\begin{aligned}z^2 &= (3-4\mathrm{i})^2\\ &= 9-24\mathrm{i}+16\mathrm{i}^2\\ &= 9-24\mathrm{i}-16\\ &= -7-24\mathrm{i}\end{aligned}$$

(e) $\overline{zw} = \overline{(8+31\text{i})}$
$= 8 - 31\text{i}$

(f) $\bar{z}\bar{w} = (3+4\text{i})(-4-5\text{i})$
$= -12 - 15\text{i} - 16\text{i} - 20\text{i}^2$
$= -12 - 31\text{i} + 20$
$= 8 - 31\text{i}$

(g) $q = \text{Re}(\bar{w}) + \text{Im}(\bar{z})\text{i}$
$= \text{Re}(-4-5\text{i}) + \text{Im}(3+4\text{i})\text{i}$
$= -4 + 4\text{i}$

3. $$\begin{aligned}(1+\text{i})^5 &= 1 + 5(\text{i}) + 10(\text{i}^2) + 10(\text{i}^3) + 5(\text{i}^4) + \text{i}^5\\ &= 1 + 5\text{i} - 10 - 10\text{i} + 5 + \text{i}\\ &= -4 - 4\text{i}\end{aligned}$$

4. $$\begin{aligned}(1-3\text{i})^3 &= 1^3 + 3(1^2)(-3\text{i}) + 3(1)(-3\text{i})^2 + (-3\text{i})^3\\ &= 1 - 9\text{i} + 27\text{i}^2 - 27\text{i}^3\\ &= 1 - 9\text{i} - 27 + 27\text{i}\\ &= -26 + 18\text{i}\end{aligned}$$
$\therefore \quad \text{Im}\left((1-3\text{i})^3\right) = 18$

5. (a) $3 \times 2 = 6$

(b) $$\begin{aligned}\text{Re}\left((3-2\text{i})(2+\text{i})\right) &= \text{Re}(6 + 3\text{i} - 4\text{i} - 2\text{i}^2)\\ &= \text{Re}(6 + -\text{i} + 2)\\ &= 8\end{aligned}$$

6. No working required.

7. (a) No working required.

(b) $$\begin{aligned}6\,\text{cis}\,\frac{5\pi}{3} &= 6\,\text{cis}\left(\frac{5\pi}{3} - 2\pi\right)\\ &= 6\,\text{cis}\left(\frac{5\pi}{3} - \frac{6\pi}{3}\right)\\ &= 6\,\text{cis}\left(-\frac{\pi}{3}\right)\end{aligned}$$

8. (a) No working required

(b) No working required

(c) $$\begin{aligned}zw &= (8\times 2)\,\text{cis}\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)\\ &= 16\,\text{cis}\,\frac{13\pi}{12}\\ &= 16\,\text{cis}\left(\frac{13\pi}{12} - 2\pi\right)\\ &= 16\,\text{cis}\left(-\frac{11\pi}{12}\right)\end{aligned}$$

(d) Use the commutative property of multiplication and no working is needed.

(e) $$\begin{aligned}\text{i}w &= \left(\text{cis}\,\frac{\pi}{2}\right)\left(2\,\text{cis}\,\frac{\pi}{3}\right)\\ &= 2\,\text{cis}\left(\frac{\pi}{3} + \frac{\pi}{2}\right)\\ &= 2\,\text{cis}\,\frac{5\pi}{6}\end{aligned}$$

(f) $$\begin{aligned}\text{i}z &= 8\,\text{cis}\left(\frac{3\pi}{4} + \frac{\pi}{2}\right)\\ &= 8\,\text{cis}\,\frac{5\pi}{4}\\ &= 8\,\text{cis}\left(\frac{5\pi}{4} - 2\pi\right)\\ &= 8\,\text{cis}\left(-\frac{3\pi}{4}\right)\end{aligned}$$

(g) $$\begin{aligned}\frac{z}{w} &= \frac{8}{2}\,\text{cis}\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)\\ &= 4\,\text{cis}\,\frac{5\pi}{1}2\end{aligned}$$

(h) No working required.

9. The initial case, where $n = 1$,

$$\begin{aligned}\text{L.H.S.} &= 5(1 + 2^0) + 2\\ &= 12\\ \text{R.H.S.} &= 1(1+6) + 5(2^1 - 1)\\ &= 7 + 5\\ &= 12\\ &= \text{L.H.S.}\end{aligned}$$

is true.

Assume the statement is true for $n = k$, i.e.

$$\begin{aligned}&12 + 19 + 31 + 53 + \ldots + \left(5(1 + 2^{k-1}) + 2k\right)\\ &\qquad = k(k+6) + 5(2^k - 1)\end{aligned}$$

Then for $n = k + 1$

$$\begin{aligned}&12 + 19 + 31 + 53 + \ldots + \left(5(1 + 2^{k-1}) + 2k\right)\\ &\quad + \left(5(1 + 2^k) + 2(k+1)\right)\\ &\quad = k(k+6) + 5(2^k - 1) + 5(1 + 2^k) + 2(k+1)\\ &\quad = k(k+6) + 5\times 2^k - 5 + 5 + 5\times 2^k + 2k + 2\\ &\quad = k(k+6) + 5\times 2^k + 5\times 2^k + 2k + 2\\ &\quad = k(k+6) + 5\times 2\times 2^k + 2k + 2\\ &\quad = k(k+6) + 5\times 2^{k+1} + 2k + 2\\ &\quad = k(k+6) + 5\times 2^{k+1} - 5 + 5 + 2k + 2\\ &\quad = k(k+6) + 5(2^{k+1} - 1) + 2k + 7\\ &\quad = k(k+6) + 5(2^{k+1} - 1) + (k+6) + (k+1)\\ &\quad = (k+1)(k+6) + 5(2^{k+1} - 1) + (k+1)\\ &\quad = (k+1)(k+7) + 5(2^{k+1} - 1)\\ &\quad = (k+1)\left((k+1) + 6\right) + 5(2^{k+1} - 1)\end{aligned}$$

Thus if the statement is true for $n = k$ it is also true for $n = k + 1$.

Since the statement is true for $n = 1$ it follows by induction that it is true for all integer $n \geq 1$. □